Claim If sum of digits of a number add to a multiple of 9, the original number is also divisible by 9.

pf Let $x = p_n p_{n-1} \cdots p_2 p_1 p_0$ be any number, where

p_n is the digit $(0,1,\ldots,9)$ in the $(n+1)^{st}$ place value, $\ldots$, p_2 is the digit in the $10^2 = 100$ place value, p_1 = digit in 10 place value and p_0 is the 1s digit

And say we know ① $p_0 + p_1 + p_2 + \cdots + p_n = 9m$ for some whole number m.

Then x can be written in expanded form as

$$② \quad x = 10^n p_n + 10^{n-1} p_{n-1} + \cdots + 10^2 p_2 + 10 p_1 + p_0$$

but from ①, we have $p_0 + p_1 + \cdots + p_n = 9m$
$\iff$ $p_0 = 9m - p_1 - p_2 - \cdots - p_n$

$\Rightarrow$ ② becomes

$$x = 10^n p_n + 10^{n-1} p_{n-1} + \cdots + 10^2 p_2 + 10 p_1 + 9m - p_1 - p_2 - \cdots - p_n$$

$$③ \quad x = p_n(10^n - 1) + p_{n-1}(10^{n-1} - 1) + \cdots + p_2(10^2 - 1) + p_1(10 - 1) + 9m$$

but $10^n - 1 = \underbrace{99\cdots9}_{n \text{ places}}$, $10^{n-1} - 1 = \underbrace{99\cdots9}_{n-1 \text{ places}}$, $\ldots$, $10^2 - 1 = 99$

$10 - 1 = 9$

$\Rightarrow$ all of the coefficients in ③ are divisible by 9
$\Rightarrow$ x is divisible by 9. :)