Math1080 Final Review Sessions

8.6
#21

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 2 & 9 & 0 & 0 & 1 \end{array}\right] (-2)$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -4 & -2 & 1 & 0 \\ 0 & 2 & 9 & 0 & 0 & 1 \end{array}\right] (-2)$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -4 & -2 & 1 & 0 \\ 0 & 0 & 17 & 4 & -2 & 1 \end{array}\right]$$

$$(-2)\ (4) \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -4 & -2 & 1 & 0 \\ 0 & 0 & 1 & \frac{4}{17} & \frac{-2}{17} & \frac{1}{17} \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{9}{17} & \frac{4}{17} & \frac{-2}{17} \\ 0 & 1 & 0 & \frac{-18}{17} & \frac{9}{17} & \frac{4}{17} \\ 0 & 0 & 1 & \frac{4}{17} & \frac{-2}{17} & \frac{1}{17} \end{array}\right]$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad A^{-1} \frac{1}{ad-cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Ex:

$$\underset{3\times 2}{C} = \begin{bmatrix} 1 & 3 \\ 7 & 2 \\ 9 & 5 \end{bmatrix} \quad \underset{2\times 3}{C^T} = \begin{bmatrix} 1 & 7 & 9 \\ 3 & 2 & 5 \end{bmatrix}$$

Final: Fall 2012

5a) Solve

$$56 = 8e^{2x-3}$$

$$7 = e^{2x-3}$$

$$\ln 7 = \cancel{\ln e}^{2x-3}$$

$$\ln 7 = 2x-3$$

$$\frac{\ln 7 + 3}{2} = x$$

Fall 2012 Final

1c) $k(x) = (x-2)^2$

Find: $\dfrac{k(x+h)-k(x)}{h}$

$$= \frac{((x+h)-2)^2 - (x-2)^2}{h}$$

$$= \frac{[(x+h)-2][(x+h)-2] - [(x-2)(x-2)]}{h}$$

$$= \frac{\cancel{x^2} + xh - \cancel{2x} + xh - 2h - \cancel{2x} - 2h + \cancel{4} + h^2 - (\cancel{x^2} - \cancel{4x} + \cancel{4})}{h}$$

$$= \frac{2xh - 4h + h^2}{h} = 2x - 4 + h$$

Spring 2013

#2 $f(x) = 2x^3 - 5x^2 - 4x + 12$

possible rational roots:

$\pm\ 12, 6, 4, 3, 2, 1, \frac{3}{2}, \frac{1}{2}$

$$\begin{array}{r|rrrr} -2 & 2 & -5 & -4 & 12 \\ & & -4 & 18 & -28 \\ \hline & 2 & -9 & 14 & \end{array}$$

possible

$+$ roots: 2 or 0

$-$ roots: 1

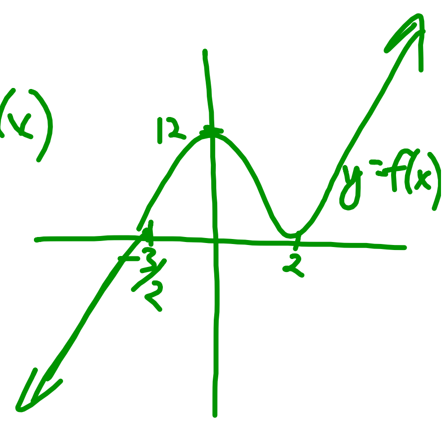
$$\begin{array}{r|rrrr} 2 & 2 & -5 & -4 & 12 \\ & & 4 & -2 & -12 \\ \hline & 2 & -1 & -6 & 0 \end{array}$$

Factors: $(x-2)(2x^2 - x - 6) = f(x)$

$(x-2)(x-2)(2x+3) = f(x)$

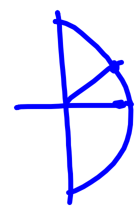
roots: $x = 2$, mult 2

$x = -\frac{3}{2}$, mult 1



Ex: 1) $\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right) = \sin^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{6}$

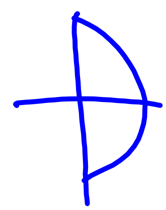


this direction does not always undo itself

2) $\sin\left(\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$

$= \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

always undoes itself in this direction



Ex $y = \frac{x-3}{x+2} = f(x)$

to find inverse:

$$x = \frac{y-3}{y+2}$$

$$(y+2)x = \frac{(y-3)}{\cancel{(y+2)}}\cancel{(y+2)}$$

$$xy + 2x = y - 3$$

$$xy - y = -2x - 3$$

$$y(x-1) = -2x - 3$$

$$y = \frac{-2x-3}{x-1} = f^{-1}(x)$$

Fall 1080
final #5c.

$\cot^2(2\beta) = \cot(2\beta)$ $\beta \in [0, 2\pi)$

$\cot^2(2\beta) - \cot(2\beta) = 0$ $0 \le \beta < 2\pi$

$\cot(2\beta)(\cot(2\beta) - 1) = 0$ $0 \le 2\beta < 4\pi$

$\cot 2\beta = 0$ $\cot 2\beta = 1$

$\frac{\cos 2\beta}{\sin 2\beta} = 0$

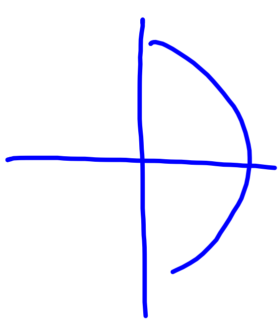
$2\beta = \pi/2, 3\pi/2, 5\pi/2, 7\pi/2, \cancel{9\pi/2}, \cancel{13\pi/2}$

$\beta = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

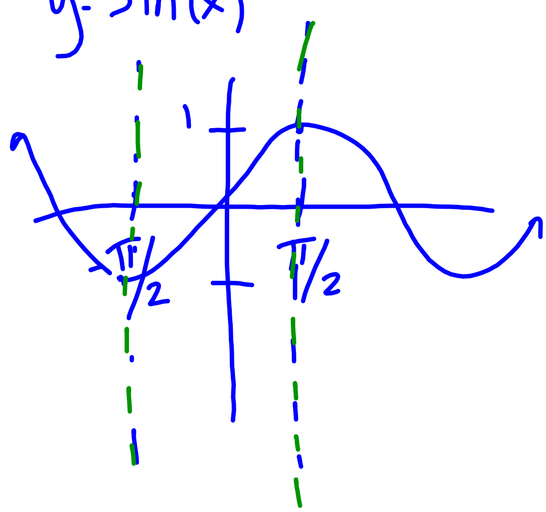
$2\beta = \pi/4, 5\pi/4, 9\pi/4, 13\pi/4$

$\beta = \pi/8, 5\pi/8, 9\pi/8, 13\pi/8$

$\sin^{-1}(x) = y$



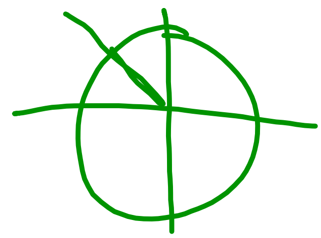
$y = \sin(x)$



Fall 1080
Final

13a) $\csc\left(\sin^{-1}\left(\frac{3}{7}\right)\right)$

$$\frac{1}{\sin\left(\sin^{-1}\frac{3}{7}\right)} = \frac{1}{\frac{3}{7}} = \frac{7}{3}$$

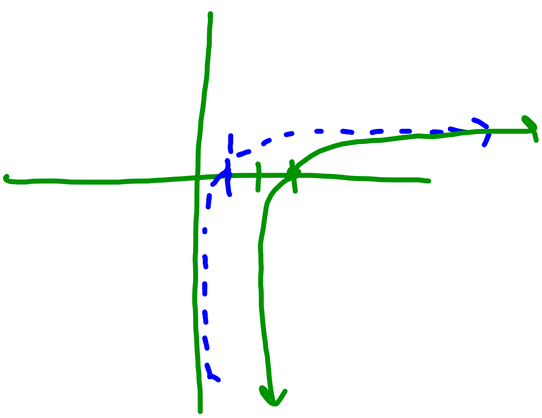


13d) $\arctan(\tan 135^\circ)$

$\tan(135^\circ) = -1$

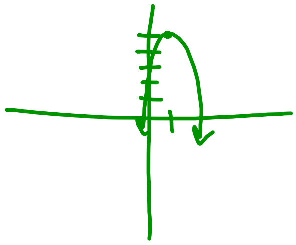
$\arctan(-1) = -45^\circ$

15b) $y = \log_5(x-2)$ $y = \log_5(x)$



ex1)

$y = -3(x-1)^2 + 5$

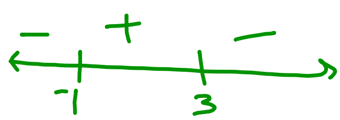


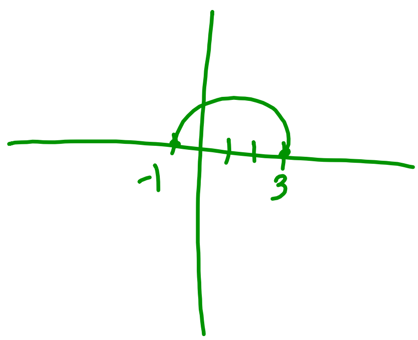
1080 S3E2

5a) graph $f(x) = \sqrt{-(x+1)(x-3)}$

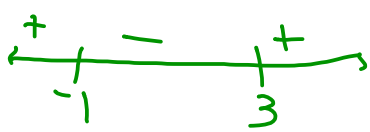
$-(x+1)(x-3) \geq 0$

Domain: $[-1, 3]$





$(x+1)(x-3) \leq 0$



III. state the domain, and solve:

$\log_4 x + \log_4 (x-3) - 1 = 0$

$x > 3$ $\quad \log_4 x(x-3) - 1 = 0$

$\log_4 (x^2 - 3x) = 1$

$4^1 = x^2 - 3x \iff 0 = x^2 - 3x - 4$

$0 = (x-4)(x+1)$

$x = 4, \cancel{-1}$

Summer final

$f(x) = \frac{x+4}{2x-1}$ $g(x) = x^2 - 6$

1e) $f \circ g(x) = f(g(x))$

$f(g(x)) = \frac{(x^2-6)+4}{2(x^2-6)-1} = \frac{x^2-2}{2x^2-13}$

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5. Solve the system of eq.

$$x + y - 3z = 9$$
$$-x + 2y = 6$$
$$x - y + z = -5$$

$$\begin{bmatrix} 1 & 1 & -3 \\ -1 & 2 & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ -5 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -3 & 9 \\ -1 & 2 & 0 & 6 \\ 1 & -1 & 1 & -5 \end{array}\right] \left[\begin{array}{ccc|c} 1 & 1 & -3 & 9 \\ -1 & 2 & 0 & 6 \\ 0 & 1 & 1 & 1 \end{array}\right]$$

$$(\tfrac{1}{3})\left[\begin{array}{ccc|c} 1 & 1 & -3 & 9 \\ 0 & 3 & -3 & 15 \\ 0 & 1 & 1 & 1 \end{array}\right] (-1)\left[\begin{array}{ccc|c} 1 & 1 & -3 & 9 \\ 0 & 1 & -1 & 5 \\ 0 & 1 & 1 & 1 \end{array}\right] (\tfrac{1}{2})\left[\begin{array}{ccc|c} 1 & 0 & -2 & 4 \\ 0 & 1 & -1 & 5 \\ 0 & 0 & 2 & -4 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 5 \\ 0 & 0 & 1 & -2 \end{array}\right] \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$x = 0$ $z = -2$
$y = 3$

1. Let $f(x) = x^2 + 2x + 3$

Find $\dfrac{f(x+h) - f(x)}{h}$

$$\frac{(x+h)^2 + 2(x+h) + 3 - (x^2 + 2x + 3)}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 + \cancel{2x} + 2h \cancel{+3} - \cancel{x^2} - \cancel{2x} \cancel{-3}}{h}$$

$$= (h^2 + 2xh + 2h)/h$$

$$= \frac{\cancel{h}(h + 2x + 2)}{\cancel{h}} = \boxed{h + 2x + 2}$$

2. a)

$$-x - 5y - 5z = 2$$
$$4x - 5y + 4z = 19$$
$$x + 5y - z = -20$$

$$\left[\begin{array}{ccc|c} -1 & -5 & -5 & 2 \\ 4 & -5 & 4 & 19 \\ 1 & 5 & -1 & -20 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 5 & -1 & -20 \\ 4 & -5 & 4 & 19 \\ 0 & 0 & -6 & -18 \end{array}\right])+ \quad \left[\begin{array}{ccc|c} 1 & 5 & -1 & -20 \\ 5 & 0 & 3 & -1 \\ 0 & 0 & -6 & -18 \end{array}\right]$$

$$x + 5y - z = -20$$
$$5x + 3z = -1$$
$$-6z = -18$$

$z = 3$
$x = -2$
$y = -3$

$(-2, -3, 3)$

2 b

$$x + 3y = -17$$
$$3x = -6$$
$$4x - 3y + 6z = 25$$

$x = -2$

$y = -5$

$z = 3$

$(-2, -5, 3)$

$$-2 + 3y = -17$$
$$3y = -15$$
$$y = -5$$

$$4(-2) - 3(-5) + 6z = 25$$
$$-8 + 15 + 6z = 25$$
$$-15 \qquad -15$$
$$+8 \qquad +8$$
$$6z = 18$$
$$z = 3$$

3. $\log_7 5x = \log_7 (2x+9)$

$7 \qquad 7$

$$5x = 2x + 9$$
$$-2x \quad -2x$$
$$3x = 9$$
$$x = 3$$

$$20 \log_6 U + 5 \log_6 V$$
$$= \log_6 U^{20} + \log_6 V^5$$
$$= \log_6 (U^{20} \cdot V^5)$$

4. find $f^{-1}(x)$ $\quad f(x) = \sqrt[3]{3x+7} - 1$

$x = \sqrt[3]{3y+7} - 1$ $\quad \dfrac{(x+1)^3 - 7}{3} = y$

$x + 1 = \sqrt[3]{3y+7}$

$(x+1)^3 = 3y + 7$

$(x+1)^3 - 7 = 3y$

$f(x) = \dfrac{3}{2x-1} + 5$ find $f^{-1}(x)$

$5 + \dfrac{3}{2y-1} = x$

$x - 5 = \dfrac{3}{2y-1}$

$(2y-1)(x-5) = 3$

$2y - 1 = \dfrac{3}{x-5}$

$\dfrac{3}{x-5} + 1 = 2y$

$\dfrac{1}{2}\left(\dfrac{3}{x-5} + 1\right) = y$

$\theta = \pi/3$

$\sin\theta = \sqrt{3}/2$

$\cos\theta = 1/2$

$\tan\theta = \sqrt{3}$

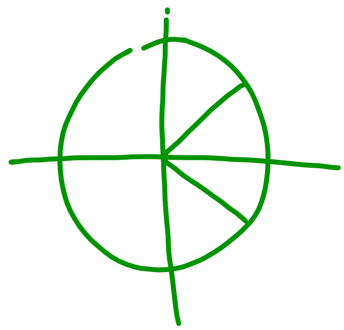
$\csc\theta = 2/\sqrt{3}$

$\sec\theta = 2$

$\cot\theta = 1/\sqrt{3}$

$\sin^{-1}\left(\frac{-\sqrt{2}}{2}\right) = \frac{-\pi}{4}$

$\cos^{-1}(0) = \frac{\pi}{2}$



Law of sines

$$\frac{\sin\beta}{b} = \frac{\sin\alpha}{a} = \frac{\sin\gamma}{c}$$

Sum Arithmetic Series

$4+8+12+16+20+\ldots\ldots+$

$d=4$

$n=200$

$$A_n = \frac{n}{2}\left(2a_1+(n-1)d\right)$$

(sum of first n terms) $A_n = \frac{n}{2}(a_1+a_n)$

nth term of sequence

$a_n = a_1 + d(n-1)$

$a_{200} = 4+4(199)$

$= 4(1+1\cdot 199)$

$= 4(200)$

$= 800$