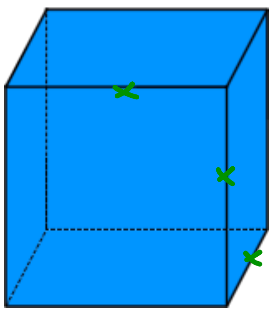
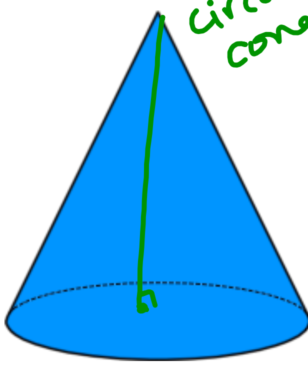
14.3 3-d Shapes/Solids

What do we call these?

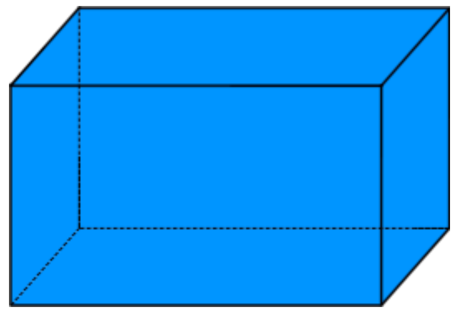
cube



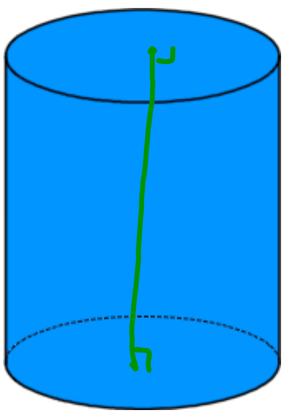
right circular cone



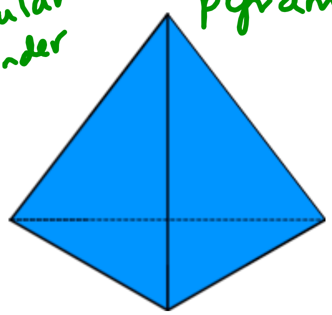
rectangular prism



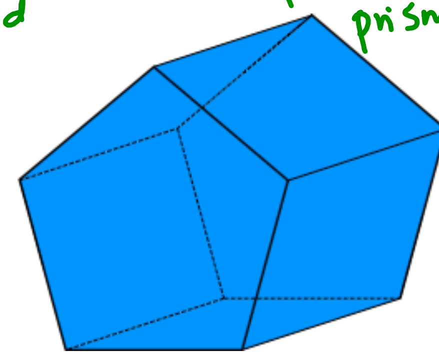
right circular cylinder

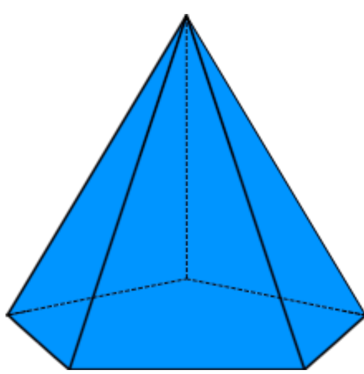


triangular pyramid



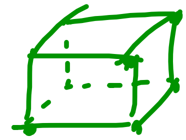
pentagonal prism





pentagonal pyramid

Basic Terminology



6 faces
12 edges
8 vertices

Face-->Polygonal region (forms dihedral angle).

Edge-->Line segment that is common to a pair of faces.

Dihedral Angle-->The angle formed by the union of polygonal regions in space that share an edge.

Vertex-->A point of intersection between edges.

Polyhedron-->(plural is polyhedra) The union of faces, any two of which have at most one edge in common, such that a connected finite region in space is enclosed without holes (i.e. such that it will contain liquid without spilling).

Convex-->A polyhedron is convex if every line segment formed by connecting two points inside the polyhedron is wholly contained inside that polyhedron OR is on a face of the polyhedron.

Types of Polyhedra

Prism-->Has two opposite, parallel faces (called *bases*) that are identical polygons.

> Right Prism-->A prism whose *lateral faces* (those faces that are neither of the bases) are rectangles; the lateral faces meet up with the bases at a right angle.

Pyramid-->Has polygon for a base and a point NOT in the plane of the base (called the apex) that is connected with line segments to each vertex of the polygonal base.

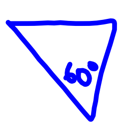
> Right Pyramid-->A pyramid whose apex lies perpendicularly over the center of the base.

Regular Polyhedron-->All faces are identical regular polygons and all dihedral angles are the same.

> Platonic Solids-->The ONLY five regular, convex polyhedra.

Semiregular Polyhedron-->Has several different regular polygonal faces, but it has the same arrangement of polygons at each vertex.

equilateral △



Other 3d Solids

Cylinder-->Has two opposite, parallel, identical, simple, closed shapes as bases and line segments that connect corresponding points from base to base (it's like a prism, except that the bases are not polygons).

Right Cylinder-->A cylinder whose "lateral" surface meets the base at right angles.

Oblique Cylinder-->A cylinder that is not a right cylinder, i.e. the "lateral" surface meets the bases at acute or obtuse angles.

Cone-->Has a simple, closed curve that creates the base and a point NOT in the plane of the base that is connected with line segments to each vertex of the base (it's like a pyramid, except that the base is not polygonal).

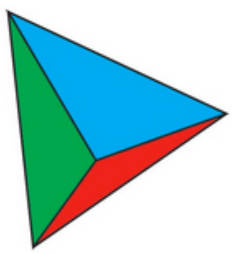
Right Cone-->A cone whose apex lies perpendicularly over the centroid of the base.

Sphere-->The set of all points in 3d space that are equidistant from a fixed point (called the *center*).

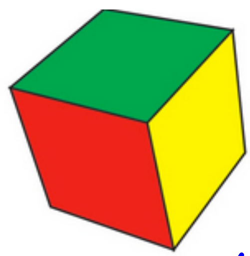
Platonic Solids

www.youtube.com/watch?v=voUVDAgFtho

www.youtube.com/watch?v=BsaOPSNMcCM

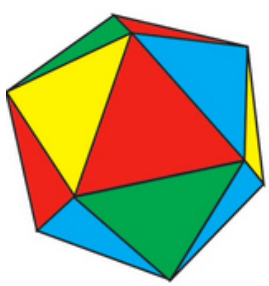


TETREHEDRON

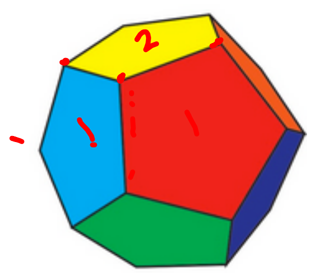


HEXAHEDRON (aka cube)

OCTOHEDRON



ICOSAHEDRON



DODECAHEDRON

$V = 5+5+3+2+2+2+1 = 20$

$E = 5+4+3+3+3+2+10 = 30$

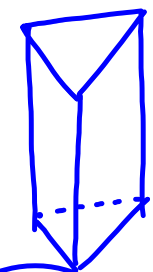
Solid Type	faces F	vertices V	edges E	Face shape
tetrahedron	4	4	6	triangle
cube	6	8	12	square
octahedron	8	6	12	triangle
dodecahedron	12	20	35	pentagon
icosahedron	20	12	35	triangle
rect. pyramid	5	5	8	rect / triangles
pentagonal pyr.	6	6	10	pentagon/triangles
pent. prism	7	10	15	pentagons/rectangles
dodeca. prism	14	24	36	dodecagons/rect.
hexag. pyr.	7	7	12	hexagon/triangles
n-gon pyramid	$n+1$	$n+1$	$2n$	n-gon/triangles
n-gon prism	$n+2$	$2n$	$3n$	n-gon/rectangles

Euler's Formula

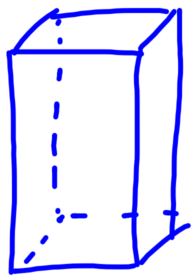
Is there some formulaic relationship between the number of faces, edges and vertices for any convex polyhedron?

$$\boxed{F + V - 2 = E}$$

14.3 A #4) min # of faces for a prism

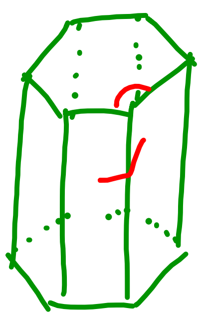


$F=5$



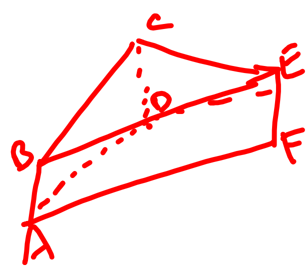
$F=6$

14.3A#11 (b)



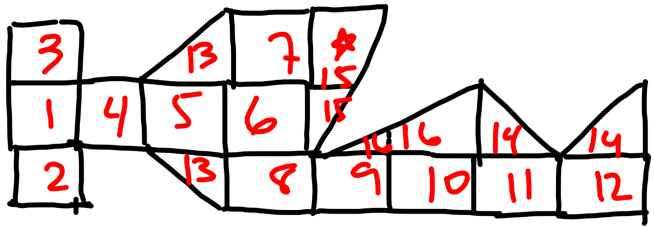
int angle of reg. hexagon = dihedral angle between lateral faces $=120°$

14.3B #3e)

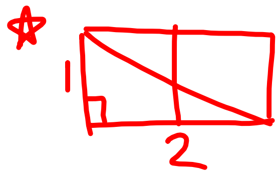


ϕ = intersection of BCE and ADF

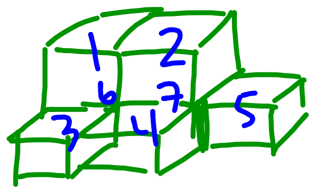
14.3 MC #12)



Area = 16 unit^2



14.3 B #2) (a)

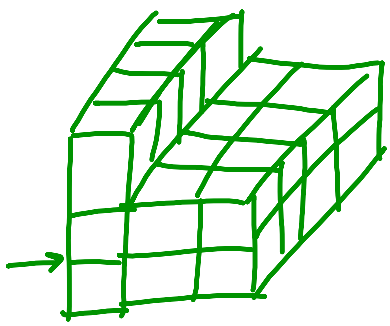


7 cubes

faces = 16

✓ 1 + 2	4 + 7
1 + 6	3 + 6
2 + 7	3 + 4
7 + 5	6 + 7

(b)



cubes

$2(2)(4) + 1(3)(4)$

$= 16 + 12 = 28$

faces glued $= 8(2) + 8(2)$

$+ 4(3)(2) + 3(3)(2)$

$+ 12(2) + 4(2)$

$= 32 + 24 + 18 + 32$

$= 64 + 42 = 106$