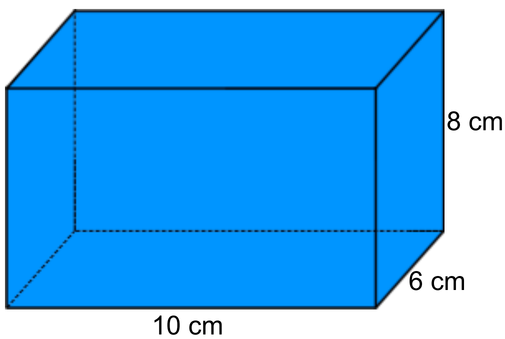
14.4 Surface Area

How do we find the surface area of a solid figure?



front/back

$SA = 2(10 \cdot 8)$

$+2(10 \cdot 6) + 2(6 \cdot 8)$

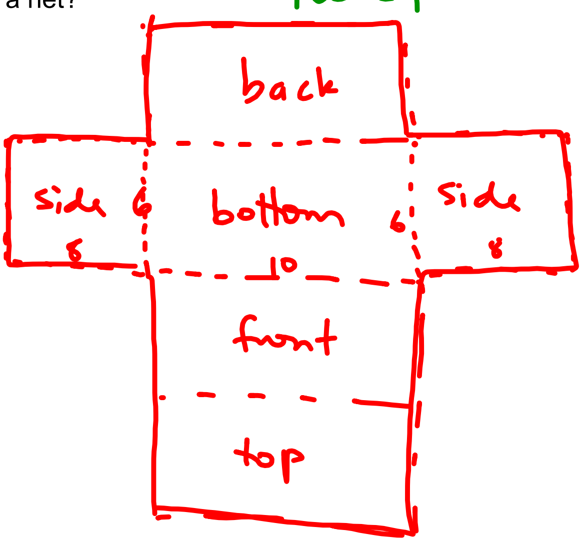
top/bottom sides

$SA = 2(80) + 2(60)$
$+2(48)$

$= 2(188) = 376$

$2(188) = 2(200-12)$
$= 400-24$

What is a net?

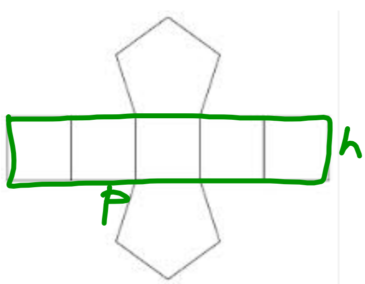


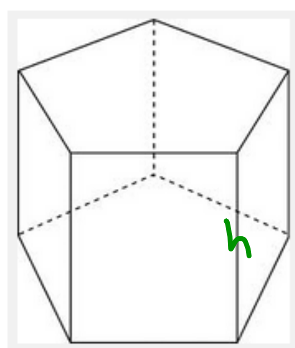
Let A = area of base

P = perimeter of base

h = height of solid

Right Prism





$\boxed{SA = 2A + Ph}$

$$SA = 2A + Ph$$

Let s = slant height

Right Pyramid

area $= \frac{1}{2} b_1 s_1$

area $= \frac{1}{2} b_2 s_2$

$$SA = A + \frac{1}{2}(b_1 s_1 + b_2 s_2 + b_3 s_3 + \dots + b_n s_n)$$

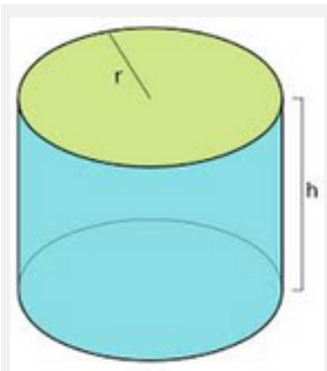
for an n-gon base

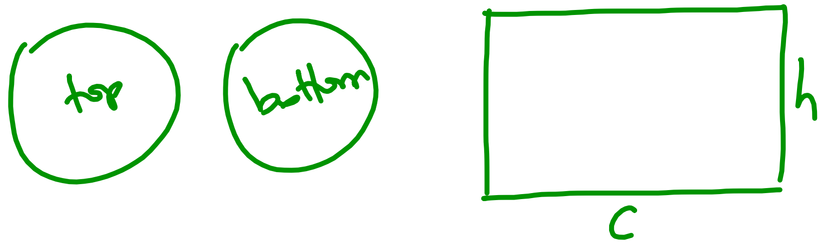
if it's a regular polygon base, $SA = A + \frac{1}{2} s(b_1 + b_2 + \dots + b_n)$

$$\boxed{SA = A + \frac{1}{2}Ps}$$

SA = A + 0.5Ps (for regular polygon base)

Right Circular Cylinder

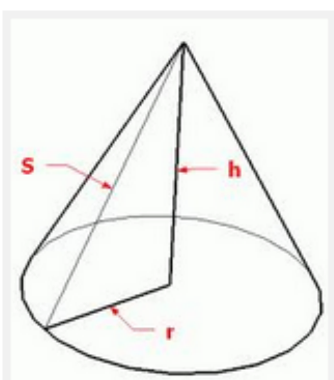




$$SA = 2A + Ch = 2\pi r^2 + 2\pi r h$$

$$\boxed{SA = 2\pi r(r+h)}$$

Right Circular Cone



$$SA = \pi r^2 + \tfrac{1}{2} C s$$

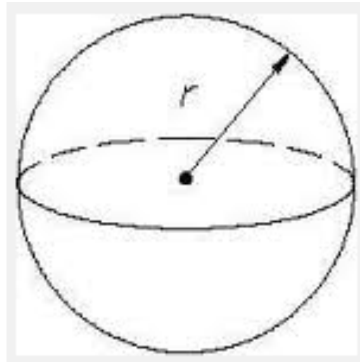
$$= \pi r^2 + \tfrac{1}{2}(2\pi r)s$$

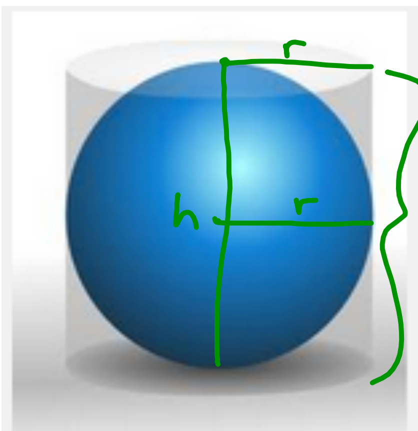
$$r^2 + h^2 = s^2$$

$$s = \sqrt{r^2 + h^2}$$

$$\boxed{SA = \pi r^2 + \pi r\sqrt{r^2 + h^2}}$$

Sphere





$h = 2r$

The Greek mathematician Archimedes discovered that the surface area of a sphere is the same as the lateral surface area of a cylinder having the same radius as the sphere and a height the length of the diameter of the sphere.

lateral SA of cylinder

$= (2\pi r)h$

$= 2\pi r(2r) = 4\pi r^2$

$= \text{SA of sphere}$

Another way to look at it is the ratio of surface area of the sphere to the entire surface area of the smallest cylinder containing the sphere is 2/3.

$\text{SA of cylinder} = 4\pi r^2 + 2\pi r^2$

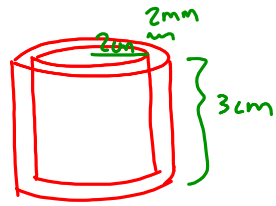
$= 6\pi r^2$

$$\frac{\text{SA sphere}}{\text{SA cyl.}} = \frac{2}{3} = \frac{\text{SA sphere}}{6\pi r^2}$$

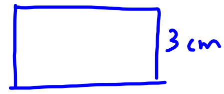
$\Rightarrow \text{SA of sphere} = \frac{2}{3}(6\pi r^2) = \boxed{4\pi r^2}$

14.4 A #4)

$2mm = 0.2cm$



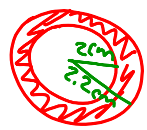
outside SA



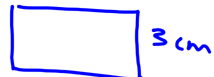
4.4π cm
$= 2\pi(2.2)$

$SA = 3(4.4\pi)\ cm^2$
$= 13.2\pi$

top view



inside



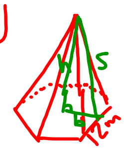
$2\pi(2) = 4\pi$
$SA = 3(4\pi) = 12\pi\ cm^2$

top/bottom "rings"

$SA = \pi(2.2)^2 - \pi(2^2)$
$= 4.84\pi - 4\pi$
$= 0.84\pi\ cm^2$

$\Rightarrow$ total $SA = 2(0.84\pi) + 13.2\pi + 12\pi \quad (cm^2)$
$= (1.68 + 13.2 + 12)\pi$
$= \boxed{26.88\pi\ cm^2} = 2688\pi\ mm^2$

14.4 A #7)



$h = 9m$ right, regular, hexagonal pyramid

A = area of base

$\boxed{SA = A + \frac{1}{2}Ps}$

P = perimeter of base

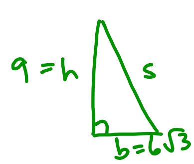
$P = 6(12) = 72\ m$

$A = \frac{3\sqrt{3}}{2}(12)^2 = 3\sqrt{3}(72) = 216\sqrt{3}\ m^2$

s = slant ht.

$b^2 + 6^2 = 12^2$
$b^2 = 144 - 36 = 108$
$b = \sqrt{108} = \sqrt{9 \cdot 12}$
$= \sqrt{9 \cdot 4 \cdot 3} = 3(2)\sqrt{3}$
$= 6\sqrt{3}$

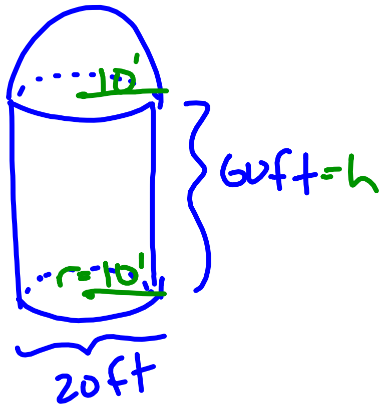
in general,
$b = \frac{1}{2}x\sqrt{3}$
where x = side length of reg. hexagon



$9^2 + (6\sqrt{3})^2 = s^2$
$81 + 108 = s^2$
$189 = s^2$
$s = \sqrt{189}$
$= \sqrt{9 \cdot 21} = 3\sqrt{21}$

$\Rightarrow SA = 216\sqrt{3} + \frac{1}{2}(72)(3\sqrt{21})$
$= (216\sqrt{3} + 108\sqrt{21})\ m^2$

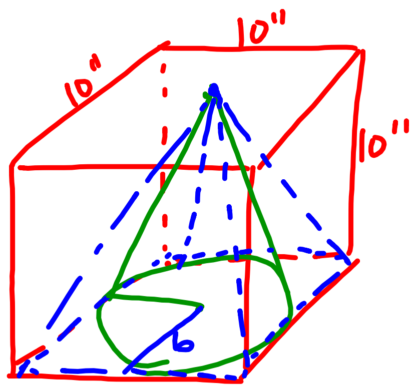
14.4 B#2b



SA = SA hemisphere + lateral SA of cylinder + SA base

$$SA = 2\pi(10)^2 + (2\pi(10))60 + \pi(10^2) = 200\pi + 1200\pi + 100\pi$$
$$= \boxed{1500\pi \text{ ft}^2}$$

14.4B#13



cone

$$SA = \pi r^2 + \pi r\sqrt{r^2+h^2}$$

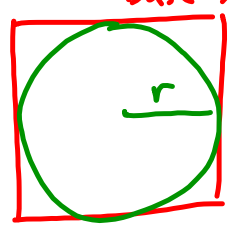
$A = 10^2$

$P = 4(10) = 40$

pyramid

$$SA = A + \tfrac{1}{2}Ps$$

base square of cube

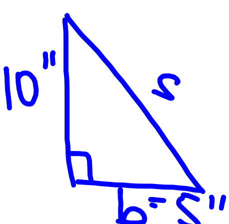


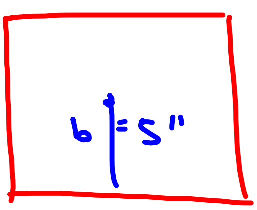
$r = 5$ in

$h = 10$ in

$$SA = 25\pi + 5\pi\sqrt{25+100}$$
$$= 25\pi + 5\pi\sqrt{25(5)}$$
$$= 25\pi + 25\pi\sqrt{5}$$
$$\boxed{SA = 25\pi(1+\sqrt{5}) \text{ in}^2}$$

cone



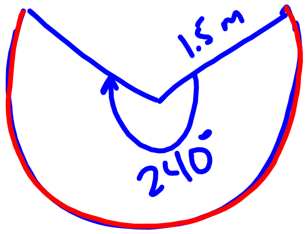


$$s^2 = 10^2 + 5^2 = 125 \Rightarrow s = 5\sqrt{5}$$

$$\Rightarrow SA = 100 + \tfrac{1}{2}(40)5\sqrt{5} = 100 + 100\sqrt{5}$$
$$= \boxed{100(1+\sqrt{5}) \text{ in}^2}$$

14.4A #12

(a)



$$SA = \left(\frac{2}{3}\right)\underbrace{\left(\pi\,(1.5)^2\right)}_{\text{area of whole circle}}$$

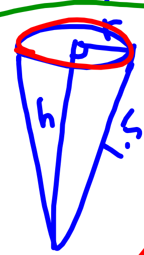
$$\frac{240°}{360°} = \frac{2}{3}$$

$$SA = \cancel{\frac{2}{3}}\pi\left(\cancel{\frac{3}{2}}\right)\left(\frac{3}{2}\right)$$

$$\boxed{SA = \frac{3}{2}\pi \ m^2}$$

(a) SA of entire cone

$= \frac{3}{2}\pi + \pi r^2$ r = radius of circle top



$C = 2\pi r$

$$s = r\theta = r\left(\frac{4\pi}{3}\right) = \frac{3}{2}\left(\frac{4\pi}{3}\right) = 2\pi$$

$$240°\left(\frac{\pi}{180°}\right) = \frac{4\pi}{3}$$

$2\pi = 2\pi r$

$r = 1$

or $\frac{2}{3}\left(2\pi\left(\frac{3}{2}\right)\right) = 2\pi$

($\frac{2}{3}$ of total start circumference)

$$SA = \frac{3}{2}\pi + \pi(1^2)$$

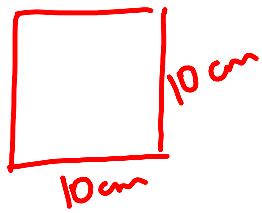
$$= \boxed{\frac{5\pi}{2} \ m^2}$$

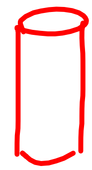
14.4A #8

SA lateral $= 2\pi r h = 2\pi(4)\left(\frac{21}{16}\right)$

$= \frac{21\pi}{2}$ in^2

14.4B #8





total SA = ?

lateral SA = 100 cm^2

$10 = 2\pi r$

$r = \frac{10}{2\pi} = \frac{5}{\pi}$

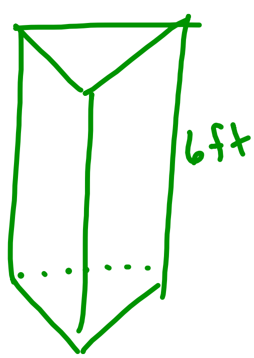
$\Rightarrow$ total $SA = 100 + 2\left(\pi\left(\frac{5}{\pi}\right)^2\right) = \left(100 + \frac{50}{\pi}\right)$ cm^2

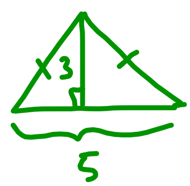
14.4B #7

$P = ?$

$A = \frac{1}{2}(3 \cdot 5)$

$= \frac{15}{2}$ ft^2

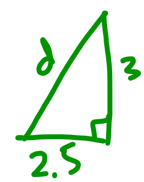




top/bottom

$h = 6$ ft

$SA = 2A + Ph$



$2.5^2 + 3^2 = d^2$

$d^2 = 6.25 + 9 = 15.25$

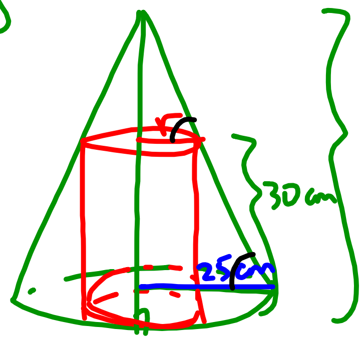
$d = \sqrt{15.25}$ ft

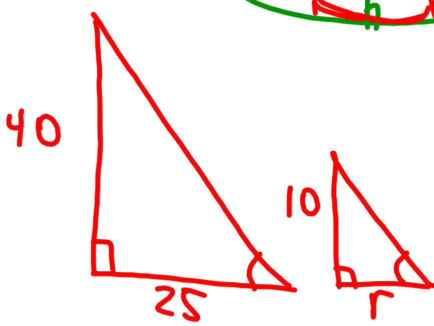
$\Rightarrow P = 2d + 5 = 2\sqrt{15.25} + 5$

$SA = 2\left(\frac{15}{2}\right) + \left(2\sqrt{\frac{61}{4}} + 5\right)6 = 15 + \frac{2(6)\sqrt{61}}{2} + 30$

$= 45 + 6\sqrt{61}$ ft^2

14.4B #19)





lateral surface area of cylinder?

$SA = 2\pi r h$

$= 2\pi r(30)$

$\frac{40}{25} = \frac{10}{r}$

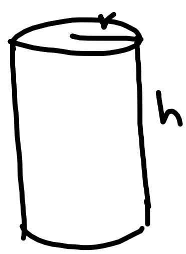
$40r = 250$

$r = \frac{25}{4}$

$SA = 60\pi\left(\frac{25}{4}\right)$

$= 375\pi \text{ cm}^2$

mc #2)



$SA = 2\pi r^2 + 2\pi r h$

double radius:

$SA = 2\pi(2r)^2 + 2\pi(2r)h$

$= \boxed{8\pi r^2 + 4\pi r h}$

double height:

$SA = 2\pi r^2 + 2\pi r(2h) = \boxed{2\pi r^2 + 4\pi r h}$