9.1 Introduction to Probability

Experiment-->The act of making an observation. ex toss a coin

Outcome-->One of the possible things that can occur. ex H or T

Sample Space-->The set of all possible outcomes (usually denoted as S). *(The sample space is called uniform if all outcomes in S are equally likely.)* ex $S=\{H,T\}$

Event-->A subset of the sample space (usually denoted as E or A). ex $A=\{T\}$

Probability-->The relative frequency we expect an event to occur (can be written as a fraction, decimal, ratio or percent)

$\boxed{P(E)=\frac{n(E)}{n(S)}}$ That is, the probability of an event E is the number of ways the event E can happen divided by the number of outcomes of S.

$n(E)$ = number of elements/outcomes in E

Since $\emptyset\subseteq E\subseteq S$, and we know that the number of elements in the empty set is 0, i.e. $n(\emptyset)=0$, then

$0\le n(E)\le n(S)$. And dividing by $n(S)$, we get

$\frac{0}{n(S)}\le\frac{n(E)}{n(S)}\le\frac{n(S)}{n(S)}$. Finally, we have

ex $n(A)=1$
$n(S)=2$
$P(A)=\frac{1}{2}$

$\boxed{0\le P(E)\le 1}$. In other words, the probability of any event is always a number between 0 and 1 (inclusive).

$P(\phi)=0$

and $P(\emptyset)=0$ and $P(S)=1$

Experimental Probability-->The probability calculated from an actual experiment. The probability for the same event may vary from observation to observation.

ex toss fair coin 100 times

get 53H, 47T $P(H)=0.53$ $P(T)=0.47$

Theoretical Probability-->This is based on ideal occurrences (usually what we mean when we say "probability").

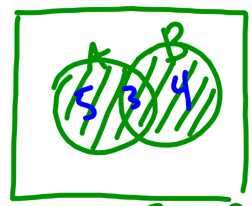
ex toss a fair coin

$P(H)=P(T)=\frac{1}{2}$

Law of Large Numbers (Bernoulli's Theorem):
If an experiment is repeated a large number of times, the experimental or empirical, probability of a particular outcome approaches a fixed number as the number of repetitions increases.
(As it turns out, the experimental mean approaches the theoretical mean.)

$P(A\cap B)=\frac{3}{12}$

ex



The probability of A ***OR*** B is equal to the probability of A plus the probability of B minus the probability of A ***AND*** B (that got counted twice).

$$\boxed{P(A\cup B)=P(A)+P(B)-P(A\cap B)}$$

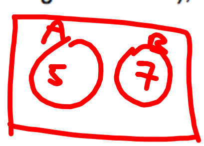
$P(A)=\frac{8}{12}=\frac{2}{3}$

$P(B)=\frac{7}{12}$

If A and B are *mutually exclusive* (meaning that they have nothing in common), then

$$P(A\cup B)=P(A)+P(B) \text{ since } A\cap B=\emptyset$$

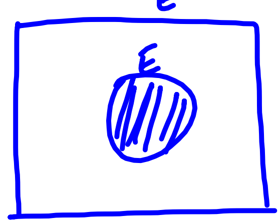
ex



The probability of the the event not happening equals 1 minus the probability of the event happening.

$$P(\bar{E})=1-P(E)$$

E



$E^c=\bar{E}$ (not E)

Ex 1 Roll a die.

(a) What is the sample space, S?

$S = \{1, 2, 3, 4, 5, 6\}$

(b) Let E = event that you roll an even number. What is P(E)?

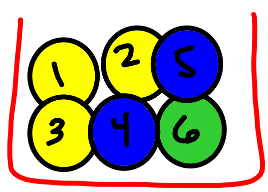
$E = \{2, 4, 6\}$

$P(E) = \frac{3}{6} = \frac{1}{2}$

$P(\bar{E}) = \frac{1}{2}$ = prob. that roll an odd #

(c) P(rolling a six) = ? $\frac{1}{6}$

Ex 2 Balls P(2 of same color)



$S = \{YY, YB, YG, BB, BG\}$ (not equally likely)

2 of same color = 2 yellow or 2 blue

$S_1 = \{Y_1Y_2, Y_1Y_3, Y_1B_4, Y_1B_5, Y_1G_6,$
$Y_2Y_3, Y_2B_4, Y_2B_5, Y_2G_6,$
$Y_3B_4, Y_3B_5, Y_3G_6,$
$B_4B_5, B_4G_6,$
$B_5G_6\}$

P(2 same color)

= P(BB or YY)

= P(BB) + P(YY)

$= \frac{1}{15} + \frac{3}{15} = \frac{4}{15}$

Ex 3 For a deck of cards

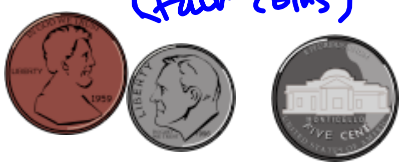
$n(S) = 52$

P(red or face card) $= P(\text{red}) + P(\text{face card}) - P(\text{red } \underline{\text{and}} \text{ face card})$

$= \frac{26}{52} + \frac{12}{52} - \frac{6}{52}$

$= \frac{32}{52} = \frac{8}{13}$

Ex 4 (fair coins)



$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Coins P(2H) or more $= P(HHH \text{ or } HHT \text{ or } HTH \text{ or } THH)$

$= P(HHH) + P(HHT) + P(HTH) + P(THH)$

$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{1}{2}$

$P(\text{exactly } 2H) = \frac{1}{2} - \frac{1}{8} = 3/8$

Ex 5

Dice P(sum 6)

2 dice thrown $S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2) \ldots \ldots (6,6)\}$

P(doubles)

$n(S) = 36$ (equally likely)

sums

$S_1 = \{2, 3, 4, \ldots, 12\}$

(not equally likely)

$P(\text{sum } 6) = P((1,5), (5,1), (2,4), (4,2), (3,3))$

$= \frac{5}{36}$

$P(\text{doubles}) = P((1,1), (2,2) \ldots, (6,6)) = \frac{6}{36} = \frac{1}{6}$

Ex 6 We have 5 skittles in a bag: 1 red, 1 purple, 1 green, 1 yellow and 1 orange.

Draw two skittles out of the bag at once.

$S = \{RP, RG, RY, RO, PG, PY, PO, GY, GO, YO\}$

Let A = event that you get one yellow skittle

$n(S) = 10$

equally likely

B = event that both skittles are the same color

C = event that neither skittle is orange

D = event that one skittle is green and the other is red

(a) List the sample space, and each of these events as a set, in list form.

$A = \{RY, PY, GY, YO\}$

$D = \{RG\}$

$B = \{\ \}$

$C = \{RP, RG, RY, PG, PY, GY\}$

$P(A) = \frac{4}{10}$

$P(C) = \frac{6}{10} = \frac{3}{5}$

$P(B) = 0$

$P(D) = \frac{1}{10}$

(b) What is the probability of each event A-D?

Ex 7

There are 10 numbered chips in a bag, numbered 0 through 9.

You pull one chip out of the bag at random.

Let A = the event that the chip is even

B = the event that the chip is a multiple of 3

C = the event that the chip is a nonzero multiple of 5

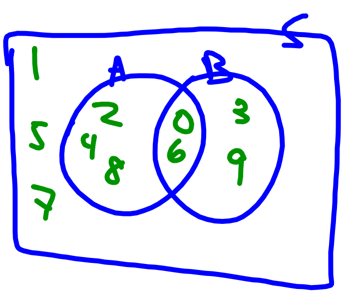
D = the event that the chip is a multiple of 4

(a) Write out the sample space and each event in set (list) format.

$S=\{0,1,2,\ldots,9\}$ $A=\{0,2,4,6,8\}$ $B=\{0,3,6,9\}$

$C=\{5\}$ $D=\{0,4,8\}$

(b) Use a Venn Diagram (for A and B) to help you determine P(A), P(B), P(not A), P(not B), P(A or B), and P(A and B).



$P(A)=\frac{5}{10}=\frac{1}{2}$

$P(B)=\frac{4}{10}=\frac{2}{5}$

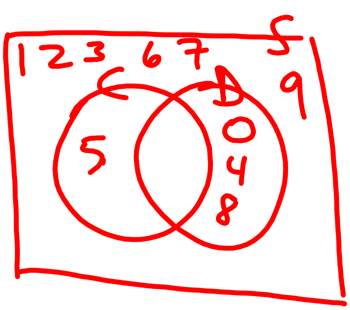
$P(\bar{A})=1-\frac{1}{2}=\frac{1}{2}$

$P(\bar{B})=\frac{3}{5}$

$P(A \text{ or } B)=\frac{7}{10}$

$P(A \text{ and } B)=\frac{2}{10}=\frac{1}{5}$

(c) Follow the instructions for (b), using events C and D instead of A and B.



$P(C)=\frac{1}{10}$

$P(D)=\frac{3}{10}$

$P(\bar{C})=\frac{9}{10}$

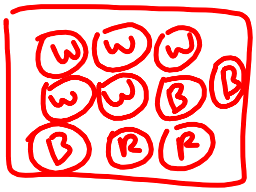
$P(\bar{D})=\frac{7}{10}$

$P(C\cup D)=\frac{4}{10}=\frac{2}{5}$

$P(C\cap D)=0$

HW ?.s

$\boxed{9.1\ A\ 13}$



(a) want $P(R)=\frac{3}{4}$

add 22 R balls

right now $P(R)=\frac{2}{10}=\frac{1}{5}$ and $P(W)=\frac{5}{10}=\frac{1}{2}$

add 1 R $\Rightarrow P(R)=\frac{3}{11}$

add 2 R $\Rightarrow P(R)=\frac{4}{12}$

$$(10+n)\,\frac{3}{4}=\left(\frac{2+n}{\cancel{10+n}}\right)\cancel{(10+n)}$$

$$\frac{3(10+n)}{4}=2+n$$

$$3(10+n)=4(2+n)$$

$$30+3n=8+4n$$

$$22=n$$

(b) add B balls & want $P(W)=\frac{1}{4}$

add 1 B $\Rightarrow P(W)=\frac{5}{11}$

add 2 B $\Rightarrow P(W)=\frac{5}{12}$

add n B $\Rightarrow P(W)=\frac{5}{10+n}=\frac{1}{4}$

$\Rightarrow n=10$

9.1 B#13) box: 5 W
3 B
2 R

add same # of balls of each color

right now $P(B) = \frac{3}{10}$

(a) want $P(B) = \frac{1}{3}$ (b) want $P(B) = 0.32$

add 1 ball of each color: $P(B) = \frac{3+1}{10+3} = \frac{4}{13}$

" 2 balls " " " : $P(B) = \frac{3+2}{10+6} = \frac{5}{16}$

⋮

" n " " " " : $P(B) = \frac{3+n}{10+3n}$

(a) $\frac{3+n}{10+3n} = \frac{1}{3}$

$3(3+n) = 10+3n$

$9+3n = 10+3n$

$9 \neq 10$

N.S.

(b) $\frac{3+n}{10+3n} = 0.32$

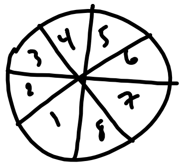
$3+n = 0.32(10+3n)$

$3+n = 3.2+.96n$

$(-100)\ -0.2 = -0.04n\ (-100)$

$20 = 4n \rightarrow n=5$

9.1A #3) (f) P(# is composite) $= \frac{3}{8}$



prime #s: 2, 3, 5, 7
mult id: 1
composite #s: 4, 6, 8

(g) P(mult. id) $= \frac{1}{8}$

B#9) (a) $P(A \cup C)$ = prob that student is taking algebra or chemistry

(b) $P(A \cap C)$ = prob that student is taking both alg & chem

(c) $1 - P(C) = P(\bar{C})$ = prob that student is not taking chem.

9.1A #5) (e) 6 Blk = K
4 Br = B
2 G

pull out 4 socks
P(2 socks of same color) = ? = 1

P(pull 4 & get no matching pair) = 0

9.1B #1d) yes

9.B #7) (a) P(win) = ? 1st 2 cards are 5 & J.

P(win) = P(3rd card is 6, 7, 8, 9, or 10) $= \frac{20}{50} = \frac{2}{5}$

(b) P(win) = ? if 1st 2 cards are 2 and K

P(win) = P(3rd card is 3, 4, 5, ..., Q) $= \frac{40}{50} = \frac{4}{5}$

9.1A #4) (b) P(face) $= \frac{12}{52} = \frac{3}{13}$ (g) P(face and club) $= \frac{3}{52}$