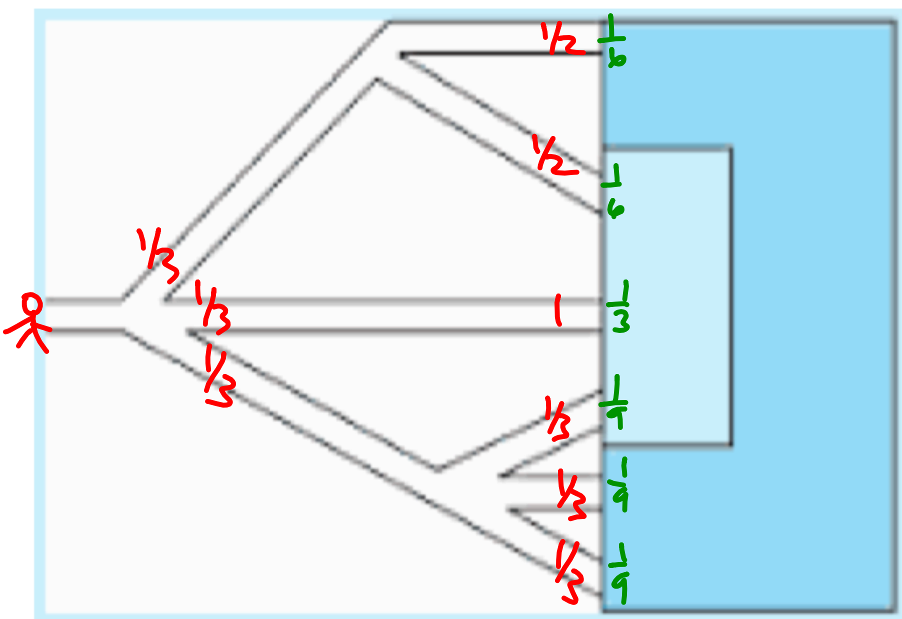
9.2 Tree Diagrams and More Probability

Lady or the Tiger?

There is a famous story, written by Frank Stockton, called "The Lady or the Tiger," which I have modified slightly to an interesting problem. It seems that the king and queen had arranged for their daughter to be married to a prince, but she fell in love with a peasant. When the king discovered this affair, he ordered that the peasant be thrown into a room full of tigers. However, in response to his daughter's pleas, he agreed to have the peasant walk through a maze to one of two rooms. The princess would be waiting in one of the rooms, and the tigers would be in the other room. The princess asked if she could choose the room in which she would wait; the kind agreed, for he believed that the chances were equal. If you were the princess, which room would you choose?



$P(\text{light blue})$

$= \frac{1}{6} + \frac{1}{3} + \frac{1}{9}$

$= \frac{3+6+2}{18}$

$= \frac{11}{18}$

$P(\text{dark blue})$

$= \frac{1}{6} + \frac{2}{9} = \frac{7}{18}$

Fundamental Counting Property-->If an event A can occur in r ways and for all of the r ways, an event B can occur in s ways, then the event A & B can occur, in succession, in rs ways. *(This can help when calculating probability.)*

Events A and B are independent if the occurrence or non-occurrence of A has absolutely no impact on B.

Probability of Independent Events: If A and B are independent, then $P(A \cap B) = P(A)P(B)$.

(Note: What is the difference between independent events and mutually exclusive events?)

Example 1: At a friend's birthday party, she had three different flavors of ice cream (vanilla, chocolate, or mint), two toppings (chocolate or strawberry sprinkles), and we could have our ice cream in a cone or a cup. I wanted to have one scoop of ice cream with a topping.

(a) How many choices did I have?

(b) If I randomly made a choice, what is the probability that I got mint ice cream with chocolate sprinkles on a cone?

(a) $\underset{\text{ice cream}}{\underline{3}}\ \underset{\text{topping}}{\underline{2}}\ \underset{\text{cup/cone}}{\underline{2}} = 12$ choices

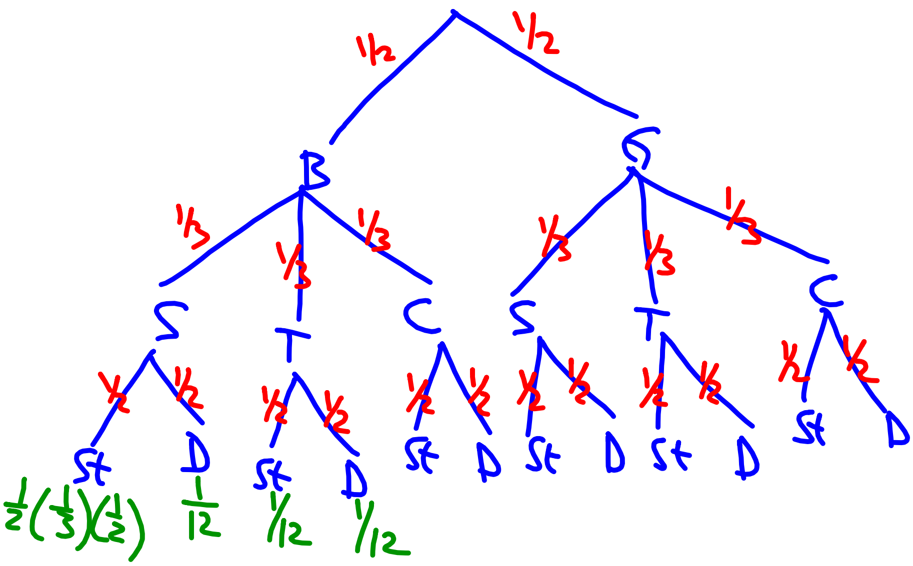
(b) $P(M, C, \text{cone}) = \frac{1}{12}$

Tree Diagram-->This can be used to represent outcomes of a multi-stage experiment (rather than writing out each element in a set).

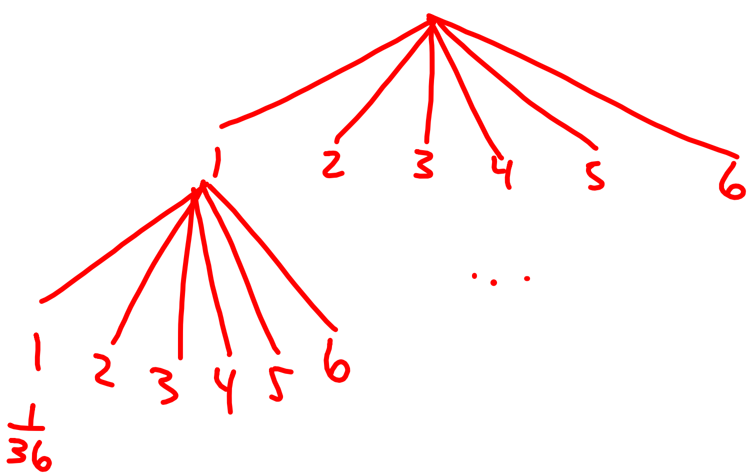
Probability Tree Diagram-->A tree diagram that has the probabilities of each event listed on the branches.

Example 2: A toy company makes tiles for a game. Each tile has these attributes--(1) it's either blue or green
(2) it's a square, triangle or circle shape
(3) it is dotted or striped.
We can use a probability tree diagram to draw all the possible different tiles that can be made with these attributes along with their probabilities.

# of possible tiles $= 2(3)(2) = 12$



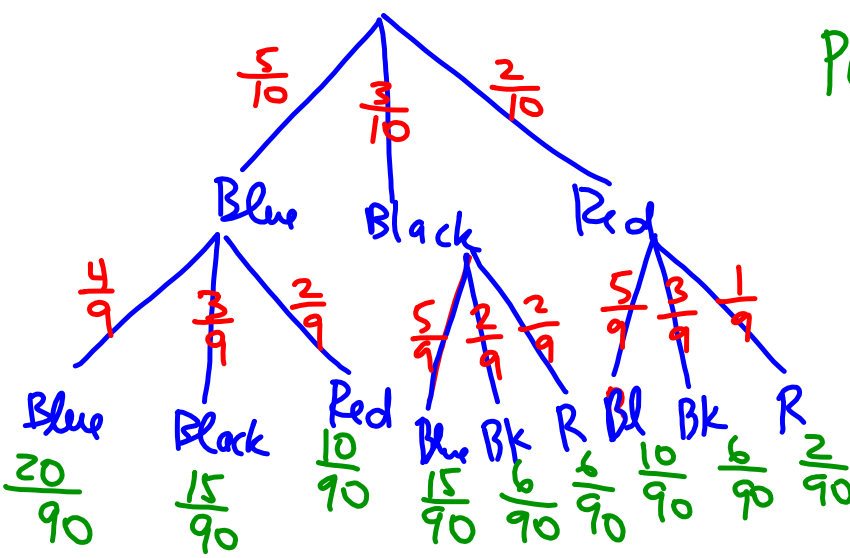
Example 3: What is the probability of getting a sum of 8 on a pair of dice?



$P(\text{sum } 8)$
$= P((2,6)(6,2)(3,5)(5,3)(4,4))$
$= \frac{5}{36}$

Example 4: I have a drawer with loose socks. In total, I have 5 blue socks, 3 black socks and 2 red socks. I pull one sock from the drawer, at random, and then another sock, trying to find a match.
Draw a probability tree diagram that represents this experiment.

(w/o replacement)



P(matched pair)

$= P(BlBl, BkBk, RR)$

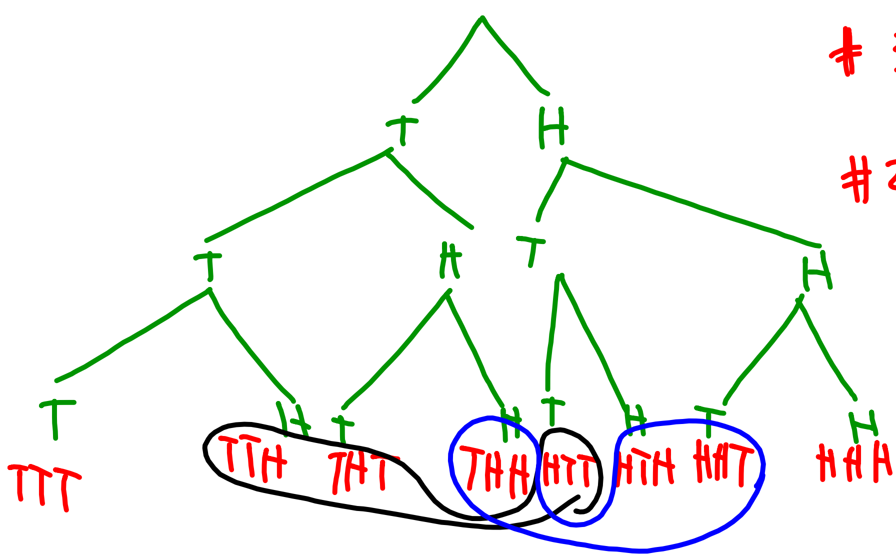
$= \frac{20}{90} + \frac{6}{90} + \frac{2}{90}$

$= \frac{28}{90} = \frac{14}{45}$

Pascal's Triangle-->

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

Example 5: Draw a tree diagram to represent tossing a coin 5 times (and recording whether it lands on Heads or Tails).
What does this have to do with Pascal's Triangle?



3T possibilities = 1
2T possibilities = 3
1T possibilities = 3
0T possibilities = 1

<u>Additive Property of Probability Tree Diagrams</u>-->For pairwise mutually exclusive events (events that have nothing in common with one another) $E_1, E_2, E_3, \ldots, E_n$ (all from the same sample space S),

if $E = E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n$,

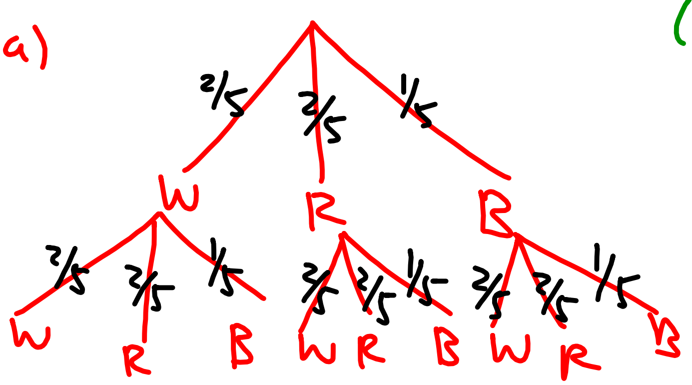
then $P(E) = P(E_1) + P(E_2) + P(E_3) + \ldots + P(E_n)$.

That is, we can add probabilities at the end of the probability tree diagram to get the probability of the event we want.

<u>Multiplicative Property of Probability Tree Diagrams</u>-->If an experiment consists of a sequence of simpler experiments and can be represented by a probability tree diagram, then the probability of any of the simpler experiments is the product of all probabilities on its branch.

Example 6: We have five candies in a jar—2 white, 2 red, and 1 blue. We pull out one candy, record its color, replace it, then draw another candy and record its color.
(a) Draw a probability tree diagram for this experiment.
(b) What is the probability that both candies are the same color?
(c) What is the probability that the first candy is white and the second candy is red?

(a)



(b) $P(\text{same color})$

$= P(WW \text{ or } RR \text{ or } BB)$

$= \frac{2}{5}\left(\frac{2}{5}\right) + \frac{2}{5}\left(\frac{2}{5}\right) + \frac{1}{5}\left(\frac{1}{5}\right)$

$= \frac{4+4+1}{25} = \frac{9}{25}$

(c) $P(WR) = \frac{2}{5}\left(\frac{2}{5}\right) = \frac{4}{25}$

Example 7: Refer to Example 4--I have a drawer with loose socks. In total, I have 5 blue socks, 3 black socks and 2 red socks. I pull one sock from the drawer, at random, and then another sock, trying to find a match.

(a) What is the probability that I choose a blue pair of socks?
(b) What is the probability that I choose a matching pair of socks?
(c) What is the probability that I choose a red and a blue sock?

(a) $P(\text{Blue Blue}) = \frac{20}{90} = \frac{10}{45} = \frac{2}{9}$

(b) $P(\text{match}) = \frac{28}{90}$

(c) $P(\text{R Bl or Bl R}) = P(\text{Bl R}) + P(\text{R Bl})$

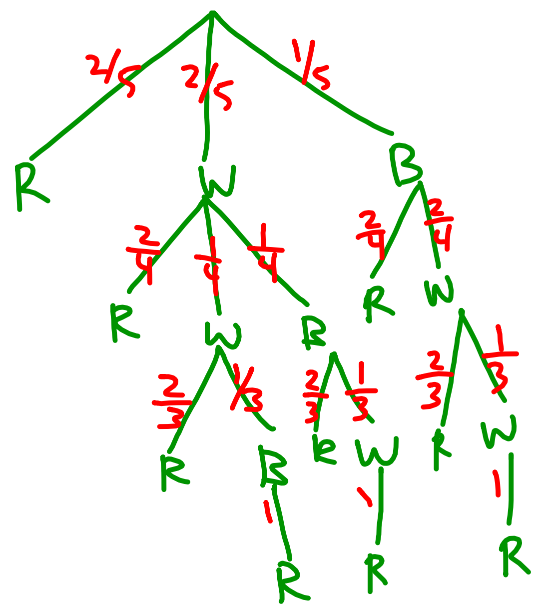
$= \frac{10}{90} + \frac{10}{90} = \frac{2}{9}$

Example 8: Refer to Example 6—We have the same candies in a jar (2 white, 2 red, and 1 blue), but this time, we will not replace the first candy we draw. End the experiment as soon as a red candy is drawn.
(a) Draw a probability tree diagram for this experiment.
(b) Given event A = only 1 draw is needed to get a red candy
event B = two draws are needed to get a red candy
event C = three draws are needed to get a red candy
1. Find P(A).
2. Find P(B).
3. Find P(C).
4. Find P(A or B). In words, what is this asking for?

$P(A) = \frac{2}{5}$

$P(B) = P(WR \text{ or } BR)$
$= \frac{2}{5}\left(\frac{2}{4}\right) + \frac{1}{5}\left(\frac{2}{4}\right)$
$= \frac{2}{10} + \frac{1}{10} = \frac{3}{10}$

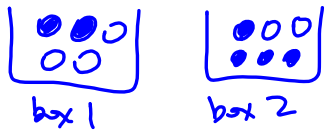
(a)



$P(A \text{ or } B)$
$= P(1 \text{ or } 2 \text{ draws to get } R)$
$= P(A) + P(B)$
$= \frac{3}{10} + \frac{2}{5} = \frac{7}{10}$

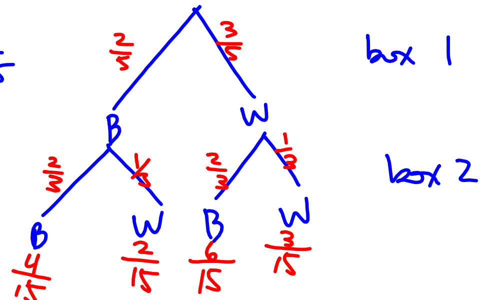
$P(C) = P(WWR \text{ or } WBR \text{ or } BWR)$
$= \frac{2}{5}\left(\frac{1}{4}\right)\left(\frac{2}{3}\right)$
$+ \frac{2}{5}\left(\frac{1}{4}\right)\left(\frac{2}{3}\right)$
$+ \frac{1}{5}\left(\frac{2}{4}\right)\left(\frac{2}{3}\right)$
$= \frac{4+4+4}{60}$
$= \frac{12}{60} = \frac{1}{5}$

9.2B#6)



$S = \{(B,B), (B,W), (W,B), (W,W)\}$

(a) $P(WW) = \frac{3}{15}$



(b) $P(BB, BW \text{ or } WB)$

$= 1 - P(WW)$

$= 1 - \frac{3}{15} = \frac{12}{15} = \frac{4}{5}$

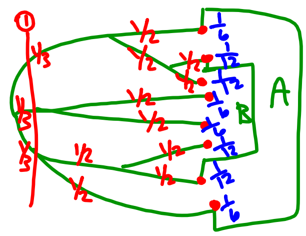
(c) $P(WW \text{ or } BW \text{ or } WB)$

$= 1 - P(BB) = 1 - \frac{4}{15} = \frac{11}{15}$

(d) $P(BW \text{ or } WB) = \frac{2}{15} + \frac{6}{15} = \frac{8}{15}$

check $\frac{2+1+1+2+2+1+1+2}{12} = \frac{12}{12}$ ✓

9.2B#17)

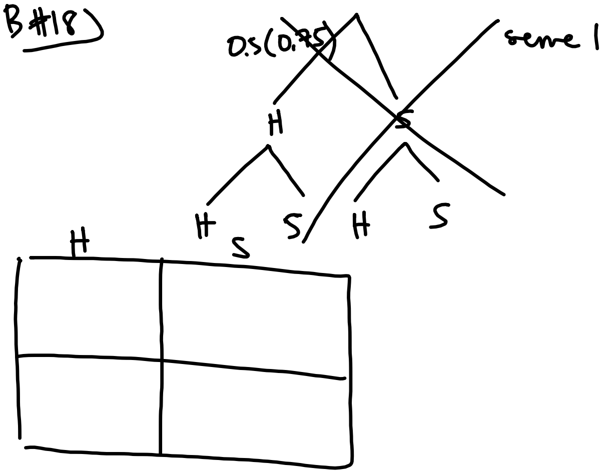


$P(A) = \frac{1}{6} + \frac{1}{12} + \frac{1}{12} + \frac{1}{6}$

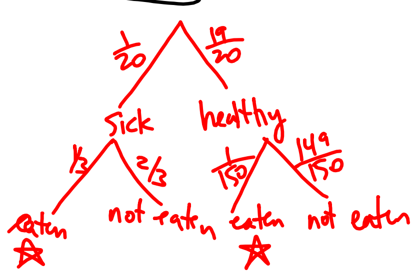
$= \frac{1}{2}$

$P(B) = \frac{1}{2}$

B#18)



19)



$P(\text{eaten}) =$

$\frac{1}{20}\left(\frac{1}{3}\right) + \frac{19}{20}\left(\frac{1}{150}\right)$

$= \frac{1}{20}\left(\frac{1}{3} + \frac{19}{150}\right)$

$= \frac{1}{20}\left(\frac{69}{150}\right)$

$= \frac{23}{20(50)}$

$= \frac{23}{1000} = 0.023$

$= 2.3\%$

B#3) P(at least 3 H on a 4 coin toss)

$$= P(3H \text{ or } 4H)$$

$$= \frac{4}{2^4} + \frac{1}{2^4} = \frac{5}{16}$$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1 — 4-coin toss

#ways to get 0H, 1H, 2H, 3H, 4H

A#10) (a) P(Paxson loses all 4 games)

$$= \frac{2}{3}\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{16}{81}$$

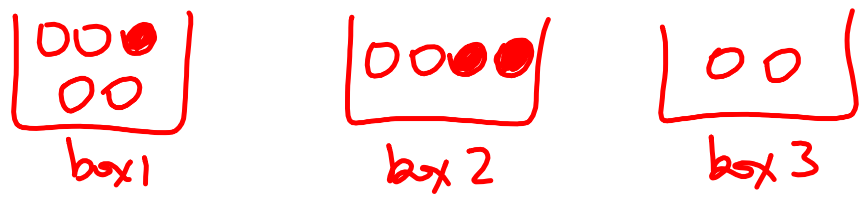
(b) P(each school wins 2 games)

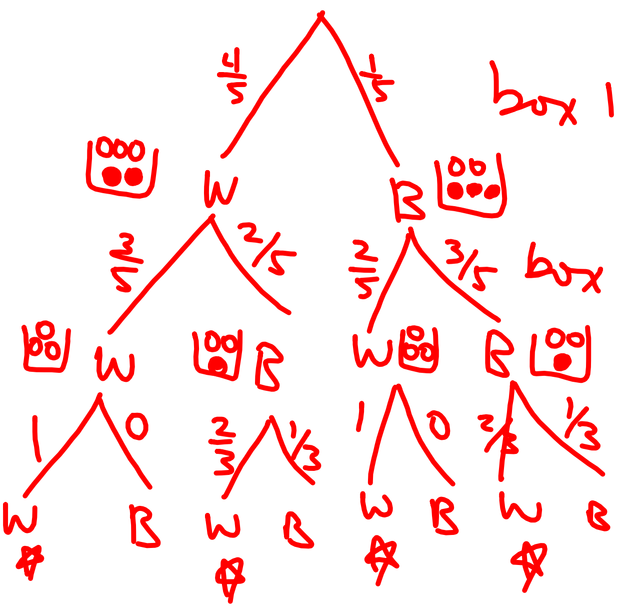
$$= 6\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^2 = \frac{\cancel{6}^{\,2}(4)}{\cancel{9}_{\,3} \cdot 9}$$

$$= \frac{8}{27}$$

List:
(PPRR, PRPR,
PRRP, RRPP,
RPRP, RPPR)

A#4)





box 1

box 2

box 3

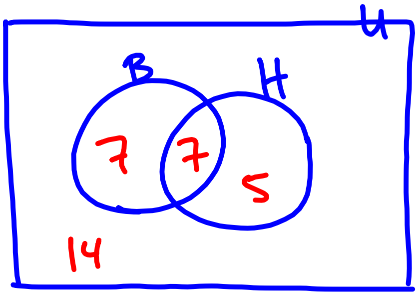
(a) $P(\text{last ball W})$

$= \frac{4}{5}\left(\frac{3}{5}\right) + \frac{4}{5}\left(\frac{2}{5}\right)\left(\frac{2}{3}\right)$

$+ \frac{1}{5}\left(\frac{2}{5}\right) + \frac{1}{5}\left(\frac{3}{5}\right)\left(\frac{2}{3}\right)$

$P(\text{last ball W}) = P(\text{WBW or WWW or BWW or BBW})$

wkst 2)



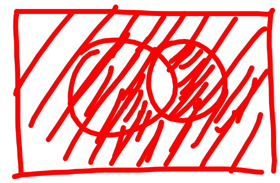
U 33

B 14

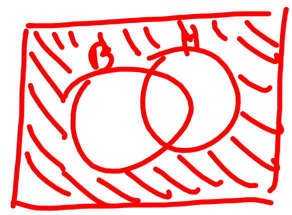
12 H

7 $H \cap B$

$P(\bar{H} \cup \bar{B}) = ?$



$\overline{H \cup B}$



1 1

1 2 1

1 3 3 1

1 4 6 4 1

0T 1T 2T 3T 4T

$P(A) = \frac{6}{16} = \frac{3}{8}$

$P(B) = \frac{1+6+1}{16} = \frac{1}{2}$

$P(C) = \frac{1}{16}$

$P(D) = P(1H \text{ or } 4H)$

$= \frac{4}{16} + \frac{1}{16} = \frac{5}{16}$