1.4 Combinations of Functions

$\in$ element of
$\forall$ for all

Adding and Subtracting Fns

Given fns $f(x)$ and $g(x)$, the sum $(f+g)(x) = f(x)+g(x)$ and the difference $(f-g)(x) = f(x)-g(x)$, $\forall x \in$ domain of both f and g.

Ex 1 Given $f(x) = 2x^2+3$ and $g(x) = 5+2x$, find $(f+g)(x)$ and $(f-g)(x)$.

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Multiplying and Dividing Fns

Given fns $f(x)$ and $g(x)$, the product $(fg)(x) = f(x)g(x)$ and the quotient $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ $\forall x \in$ domain of both f and g and such that $g(x) \neq 0$.

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1.4 (cont)

Ex 2 For $f(x) = x^2 - 1$, $g(x) = x^2 - 2x + 5$, find $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$.

Graphical Characteristics of function combinations

- x-intercepts of $(fg)(x)$ are x-intercepts of $f(x)$ and $g(x)$.
- " " $\left(\frac{f}{g}\right)(x)$ " " " " that are not x-ints. of $g(x)$
- $\left(\frac{f}{g}\right)(x)$ undefined for x-intercepts of $g(x)$

What else? •

1.4 (cont)

Function Composition

Given fns $f(x)$ and $g(x)$, the composite fn

$(f \circ g)(x) = f(g(x))$ (read "f of g of x")

☆☆☆ domain of $(f \circ g)(x)$ is all $x \in$ domain of g such that $g(x)$ is in domain of $f(x)$.

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Ex 3 For $f(x) = \sqrt{3x-12}$ and $g(x) = 2x^2+1$,

find: (a) $(f \circ g)(x)$

(b) $(g \circ f)(x)$

Ex 4 Given

x	$f(x)$	$g(x)$
-3	-1	1
-2	2	2
-1	-3	3
0	3	1
1	-2	2
2	0	3
3	5	1

Find (a) $(f \circ g)(x)$ and (b) $g(f(x))$

1.5 Transformations of Functions

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Quadratic Example

$$f(x) = a\,(b\,(x-h))^2 + k$$

vertical shift (k>0 up, k<0 down)

horizontal shift (h>0 right, h<0 left)

horizontal stretch/reflection

vertical stretch/reflection

addition/subtraction: shift

multiplication/division: stretch, reflection (if negative)

Order to perform transformations:

1. stretch
2. Reflect
3. Shift

Translation/Shift

$y = f(x-h)+k$

horizontal shift (h > 0 right, h<0 left)

vertical shift (k>0 up, k<0 down)

Reflection

(a) $y = f(-x)$

horizontal reflection (across y-axis)

(b) $y = -f(x)$

vertical reflection (across x-axis)

Stretch/Shrink $\boxed{h>0}$

(a) $y = h\,f(x)$

vertical stretch/shrink (h>1 stretch, 0<h<1 shrink)

(b) $y = f(hx)$

horizontal stretch/shrink (h>1 shrink, 0<h<1 stretch)

3.6 Transformations of Graphs

TYPES OF TRANSFORMATIONS TO $y = f(x)$

(Assume c is a constant such that $c \in \mathbb{R}$, $c > 0$.)

(For all examples in the last column of this table, we'll use $y = f(x) = x^2$ as the base or parent function graph.)

1. Shift: $h(x) = f(x) \pm c$	Shifts graph up or down by c units (if we add c, shift up; if we subtract c, shift down)	$y = x^2 + 2$ shifts graph 2 units up
$h(x) = f(x \pm c)$	Shifts graph left or right by c units (if we add c, shift left; if we subtract c, shift right)	$y = (x - 3)^2$ shifts graph 3 units right
2. Reflection: $g(x) = -f(x)$	Reflects graph vertically (across x-axis)	$y = -x^2$ reflects graph vertically
$g(x) = f(-x)$	Reflects graph horizontally (across y-axis)	$y = (-x)^2$ reflects graph horizontally
3. Stretch/Shrink: $k(x) = cf(x)$	Stretches/shrinks graph vertically (if $c > 1$, it's a stretch; if $0 < c < 1$, it's a shrink)	$y = 5x^2$ stretches graph vertically by factor of 5
$k(x) = f(cx)$	Stretches/shrinks graph horizontally (if $c > 1$, it's a shrink; if $0 < c < 1$, it's a stretch)	$y = (4x)^2$ shrinks graph horizontally by one quarter

Note: ALL the vertical effects or changes to the graph appear "outside" the function, that is, outside the base or parent function that defines the overall shape. ALL the horizontal effects or changes to the graph appear "inside" the function, that is, before we perform the essence of the function. In the above examples, the main shape of the graph is the parabola given by $y = f(x) = x^2$. So, any algebraic change that happens before we square anything is "inside" the function, and any change that happens after the square is "outside" the function.

Note also that all the vertical shifts, stretches and shrinks are "intuitive," meaning that they're as expected. Adding two outside the function, for example, shifts the graph up, and we would expect a positive vertical shift to be up. Also, the horizontal shifts, stretches and shrinks are all "counter-intuitive," meaning that they're the opposite of what we'd expect. For example, adding three inside the function shifts the graph to the left by three units, which is perhaps the opposite of what one would expect from a positive horizontal change. Adding three inside the function means a smaller x-value is needed to produce the same y-value as before the shift. Because a smaller x-value is needed, we shift left instead of right.

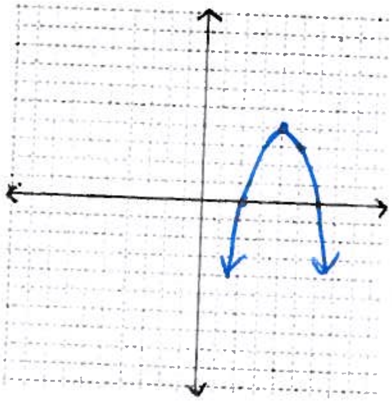
Keep in mind that we can always use the default, back-up strategy of plotting lots of points and connecting the dots to graph any function. This method of understanding the transformations of graphs, along with the knowledge of the basic function graph shapes, is simply a more powerful and sophisticated method of graphing than plotting points.

Let's do several more examples to practice.

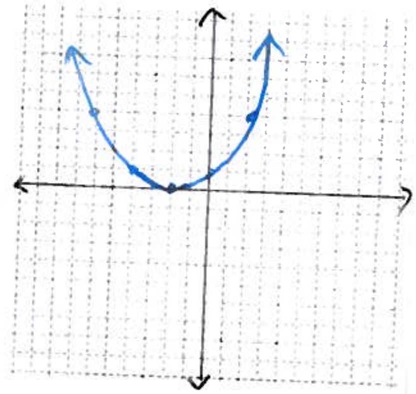
1.5 (cont)

Ex1 Given base graph $y=x^2$, write function for each graph, in form $y=C(x-h)^2+k$.

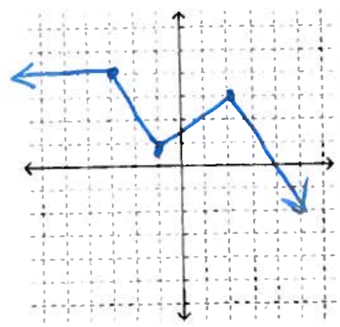
(a)



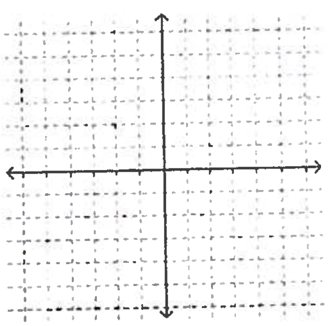
(b)



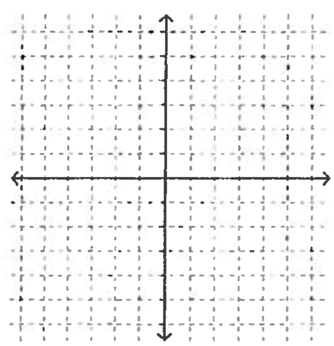
Ex2 Given this graph for $y=f(x)$, graph $g(x)=y$.



(a) $g(x)=-f(x+1)-3$



(b) $g(x)=2f(x+3)$



1.5 (cont)

EX3 Write the fn (algebraically) as described.

(a) The graph of $y=f(x)$ is shifted down 3 units, reflected across x-axis and stretched horizontally by a factor of 5.

(b) The graph of $y=f(x)$ is reflected across y-axis, shrunk horizontally in half, stretched vertically by factor of 4 and shifted left 7.

Ex4 Describe all transformations of $g(x)$, compared to base graph of $y=f(x)$.

(a) $f(x)=\sqrt{x}$,

$$g(x)=3\sqrt{\tfrac{1}{2}(x+1)}-4$$

(b) $f(x)=x^3$,

$$g(x)=-\tfrac{1}{3}(2x+4)^3$$

1.6 Quadratic Fns

☆☆☆ Quadratic Fn

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1.65 ① Standard Form: $f(x) = ax^2 + bx + c$, $a \neq 0$

 $a > 0$  $a < 0$ vertex at $\left(-b/2a,\ f(-b/2a)\right)$

② transformation Form: $f(x) = a(x-h)^2 + k$, $a \neq 0$
vertex at (h, k)

Properties of graph of $y = ax^2 + bx + c = a(x-h)^2 + k$

- y-intercept at $(0, c)$
- slope of line tangent to graph of $y = f(x)$ at $(0, c)$ is b
- x-coord of vertex positive when $ab < 0$
  negative when $ab > 0$

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- x-intercepts:
  (a) one x-int. when $k = 0 \iff b^2 = 4ac$
  (b) 2 x-ints. when $ak < 0 \iff b^2 > 4ac$
  (c) no x-ints. when $ak > 0 \iff b^2 < 4ac$.

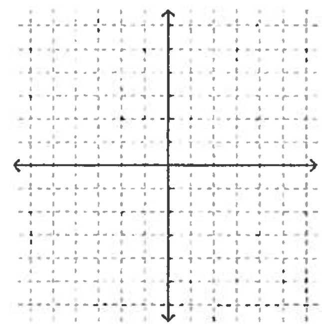
- Quadratic Formula help:
  x-intercepts are $(x_i, 0)$ where $x_i = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(☆ explain why vertex formula works for standard form)

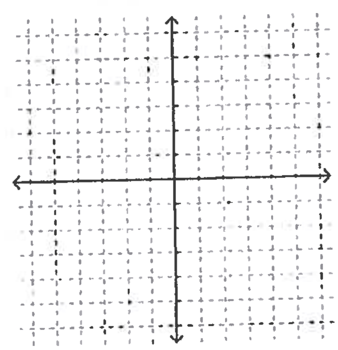
1.6 (cont)

Ex 1 Complete the square to find transformation form, and then graph.

(a) $g(x) = x^2 + 8x - 3$



(b) $g(x) = \frac{1}{3}x^2 - \frac{3}{2}x + \frac{7}{4}$



1.6 (cont)

Ex 2 Find number of x-intercepts for each fn.

(a) $g(x) = x^2 - 4x + 3$

(b) $p(x) = -6x^2 + 4x - 3$

Ex 3 Find x-intercepts and vertex.

(a) $m(x) = 3x^2 - 7x + 2$

(b) $h(x) = 4x^2 + 12x + 9$

Ex 4 Write the function for this parabolic graph.