3.2 Inverse Functions

3.18
3.27
3.28

One-to-one (1-1)

A fn is 1-1 on its domain if each value of $f(x)$ corresponds to exactly one value of x.

That is, f is 1-1 on its domain if $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

(it's 1-1 if each output has only one input that can get to that output)

Horizontal Line Test

If every horizontal line crosses through a fn's graph at most one time, then that fn is 1-1.

Ex 1 Show $g(x) = 5x - 7$ is 1-1 (algebraically).

[Note: $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

$\Longleftrightarrow$ $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$]

Inverse Fn

$f^{-1}(x)$ (read as "f inverse of x") is the inverse fn of f and it "undoes" what f does

If f is 1-1, then $f^{-1}(x)$ exists (uniquely) and $x = f^{-1}(f(x)) = f(f^{-1}(x))$

domain of f = range of f^{-1}

domain of f^{-1} = range of f

3.2 (cont)

Ex 2 Find $f^{-1}(x)$, if possible.

(a) $f(x) = 5x^3 - 1$

(b) $f(x) = 5x^2 - 1$

Find $f^{-1}(x)$:

Start w/ $y = f(x)$ and re-solve to get x by itself. Then you have $x = f^{-1}(y)$. If you want x to be the name of your input, then write $f^{-1}(x)$ by replacing y w/ x.

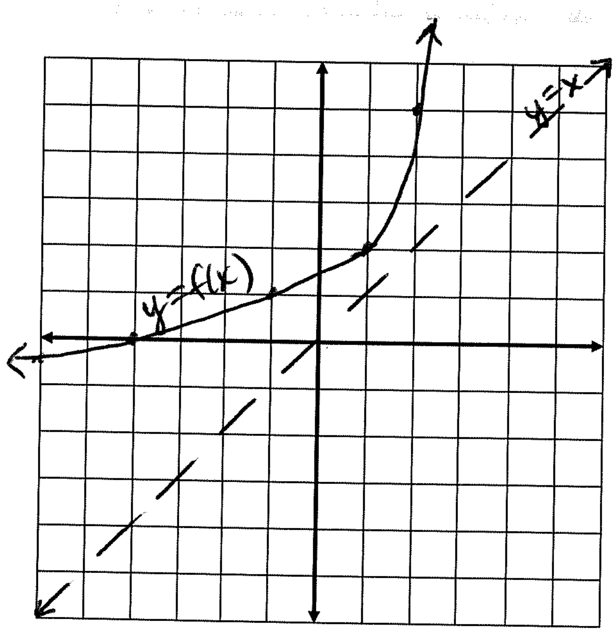
OR

Switch the x and y first. So start with $x = f(y)$. Re-solve for y. Then you have $y = f^{-1}(x)$.

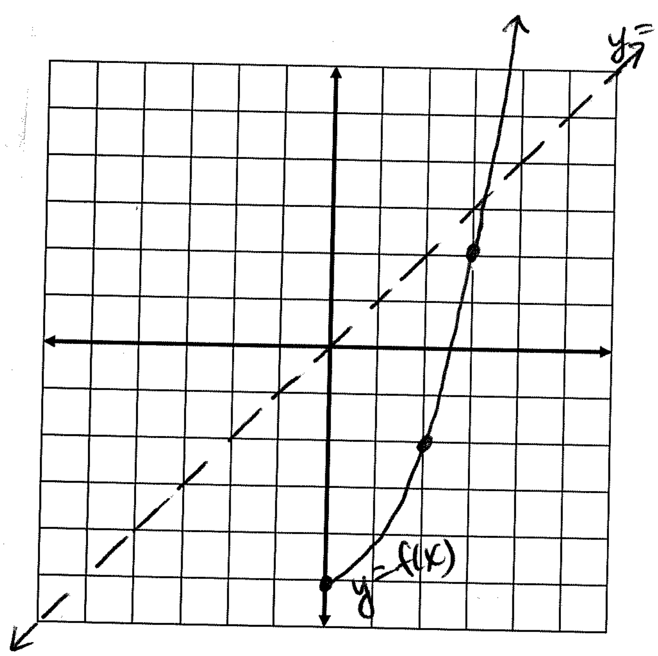
3.2 (cont)

Ex 3 Given the graph of $y=f(x)$, graph $y=f^{-1}(x)$.

(a)



(b)



3.3 Logarithmic Fns

$$\boxed{\begin{array}{l} b>0,\ b\neq 1 \\ b^x = y \iff \log_b y = x \end{array}}$$

☆☆☆ 3.30 3.31

The logarithm, base b, "undoes" the exponential with base b.

Note: we read $\log_b y = x$ as "log base b of y equals x".

Note: $\ln x = \log_e x$; $\log x = \log_{10} x$

	Domain	Range
$f(x)=b^x$	$(-\infty,\infty)$	$(0,\infty)$
$f(x)=\log_b x$	$(0,\infty)$	$(-\infty,\infty)$

Some properties

1) $e^{\ln x} = x \quad \forall\, x>0$
2) $\ln(e^x) = x \quad \forall\, x\in\mathbb{R}$
3) $b^{\log_b x} = x \quad \forall\, x>0$
4) $\log_b(b^x) = x \quad \forall\, x\in\mathbb{R}$

because $y=\log_b(x)$ and $y=b^x$ are inverse fns

Ex1 Simplify (w/o calculators)

(a) $\log_4 1$ (b) $\log_b b^8$ (c) $\log_8 64$

(d) $\log_{25}\left(\frac{1}{25}\right)$ (e) $\log_3\sqrt{3}$

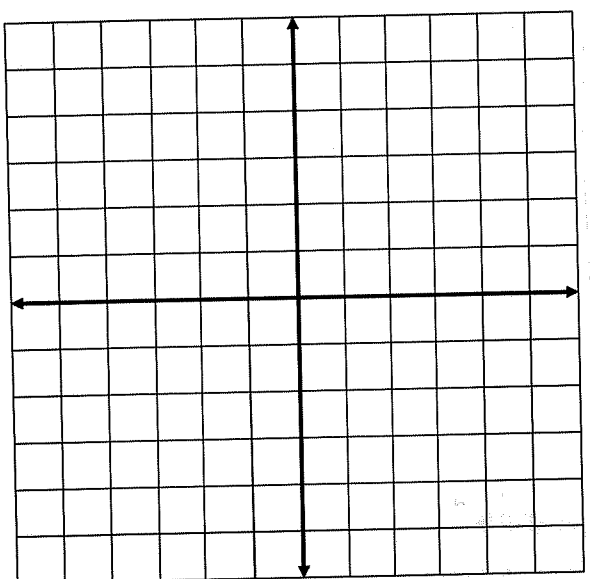
3.3 (cont)

Ex 2 Approximate (find lower and upper bounds)

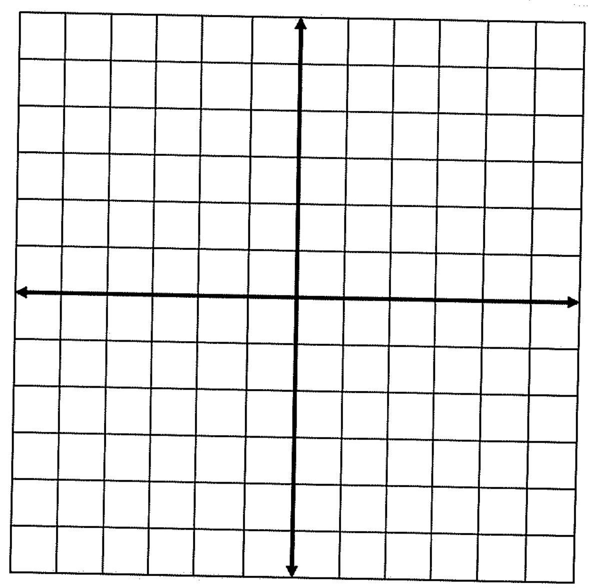
$$\log_5 100$$

Ex 3 Graph these fns.

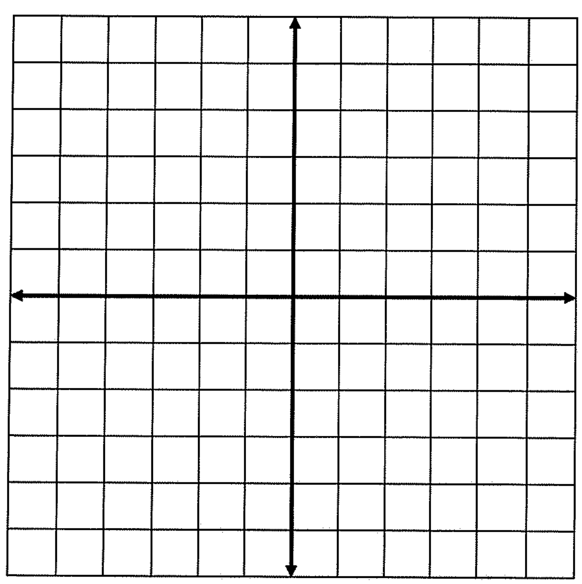
(a) $f(x) = \log_3 x$



(b) $y = -\ln(x+2) - 1$



(c) $y = \frac{1}{2}\log(2x+6)$



3.3 (cont)

Ex 4 Simplify.

(a) $\log(2^4 5^4)$

(b) $\log_5 \sqrt[3]{25} + \log_2 \sqrt[6]{4}$

Ex 5 Solve for x. $\log_6 x = 2$

3.4 Logarithmic Identities

① (Product-Sum) $\log_b(MN) = \log_b M + \log_b N$ $\quad M, N > 0$

★★★

ex $\log_3(3x) = \log_3(3) + \log_3 x$

3.34

★★★ ② (Quotient-Difference) $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$ $\quad M, N > 0$

ex $\log_5\left(\frac{4}{7}\right) = \log_5 4 - \log_5 7$

3.36

★★★ ③ (Power-Product) $\log_b(M^p) = p\log_b M$ $\quad M > 0$

ex $\log(9^5) = 5\log 9$

3.38

Note/warning: $\log_b(M^p) \neq (\log_b M)^p$

Note/warning: $\log_b(M+N)$ is not simplifiable

Ex1 Simplify.

(a) $\log_7 14 + \log_7\left(\frac{49}{2}\right)$

(b) $\log_6\left(\frac{3}{4}\right) - \log_6\left(\frac{1}{8}\right)$

Ex 2 Express as a single logarithm expression.

(a) $2\ln(x-3)+\ln(2x-1)-\ln(2x^2-7x+3)$

(b) $\frac{1}{3}\ln x+\frac{5}{3}\ln 2x-\frac{2}{3}\ln 2$

Ex 3 Use log properties/identities to expand.

(a) $\ln\left(\frac{\sqrt[3]{x^2y^5}}{w}\right)$

(b) $\log_4\left(\frac{\sqrt[4]{x}}{z^2\sqrt[3]{y^2}}\right)$

3.4 (cont)

Change of base

Let's say $f(x) = e^x$ (natural exponential fn) and

$g(x) = b^x$, $b>0$, $b \neq 1$, $b \neq e$.

Then $g(x) = b^x = (e^{\ln b})^x$ (since $e^{\ln b} = b$)

$= e^{(x \ln b)}$

which is a scaled version of $e^x = f(x)$.

3.40 $\Rightarrow g(x) = f((\ln b)x)$ ($\ln b$ is just a constant)

Likewise $h(x) = \log_b x$ can be written as scaling of

$k(x) = \ln x$.

Start w/ $y = \log_b x \iff b^y = x$ (by defn)

$\ln b^y = \ln x$

$y(\ln b) = \ln x$

$y = \dfrac{\ln x}{\ln b}$

$\Rightarrow \boxed{\log_b x = \dfrac{\ln x}{\ln b}}$ change of base formula

why do we need this?

Ex 4 Convert to natural log and to base 10 log.

(a) $\log_3 5$

(b) $\log_7 9$

3.5 Solving Exponential and Logarithmic Eqns

Strategy: (General Guidelines)

Exponential Eqns

① Isolate exponential term on one side of eqn.

② Either take log of both sides or use defn of log to rewrite.

③ Solve resulting eqn

Logarithmic Eqns

① Use log identities/properties to write logarithmic expressions in a single log term.

② Isolate log expression on one side of eqn.

③ Either exponentiate both sides (w/ same base as log) or use defn of log to rewrite

④ Solve resulting eqn.

☆ ⑤ ☆ check your answers!!!
☆

Ex 1 Solve. (give exact answers)

(a) $5 - 6e^{2x-1} = -7$

(b) $\left(\frac{1}{2}\right)^{3x} = \left(\frac{1}{4}\right)^{x+1}$

3.5 (cont)

Ex2 Solve (give exact answers)

Be careful about domain restrictions!

(a) $\log x + \log(3x-20) = 2$

(b) $\log_4(x+2) = 2 + \log_4(x-1)$

3.5 (cont)

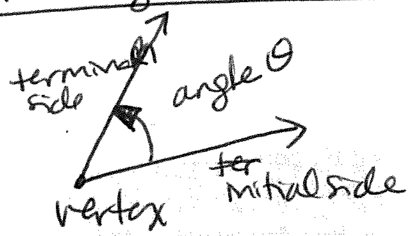
Ex 3 Solve.

(a) $4xe^{-x} = 15e^{-x} - 4x^2e^{-x}$

(b) $\log_6 x + \log_6 (x-5) = 2$

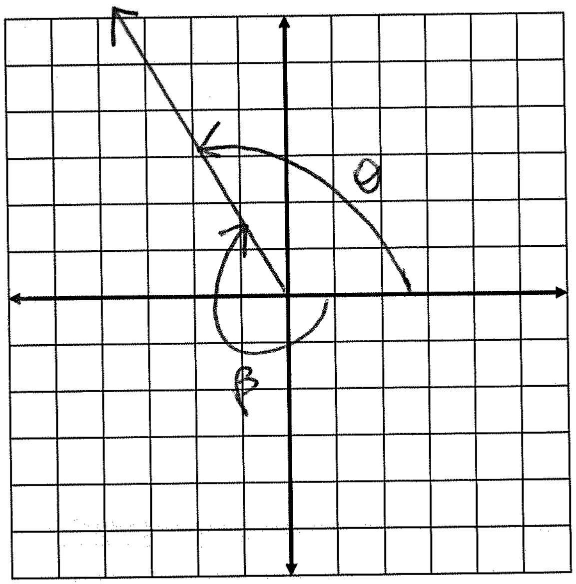
Ex 4 A drug is dissipated in the body at a rate of 12% per hour. After how many hours does the amt of drug reach only 20% of the initial dose, in the body?

4.1 Angles and Their Measures



θ is <u>positive</u> if in counterclockwise direction

θ is <u>negative</u> if in clockwise direction



☆☆☆ <u>coterminal angle</u>: β and θ
4.5
4.6 are coterminal angles because
4.4 they start & end in same places

<u>obtuse angle</u>: when it measures between 90° and 180°, exclusive.

<u>acute angle</u>: when the angle measures between 0° and 90°, exclusive

we can measure angles in degrees or radians (considered a unit-less measure)

<u>Conversions</u>: $\boxed{\pi \text{ (radians)} = 180^\circ}$

<u>Ex 1</u> convert from degrees to radians.

(a) 270°

(b) 45°

(c) 30°

Ex 2 Convert from radians to degrees.

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{5}$ (c) $\frac{-3\pi}{4}$

Ex 3 Give one positive and one negative coterminal angle for θ.

(a) $\theta = \frac{2\pi}{3}$ (b) $-36°$

Ex 4 Find the degree measure of θ.

θ is $\frac{7}{4}$ of a rotation of a full circle

Ex 5 Find the fraction of a complete rotatn that corresponds to $-135°$.

4.1 (cont)

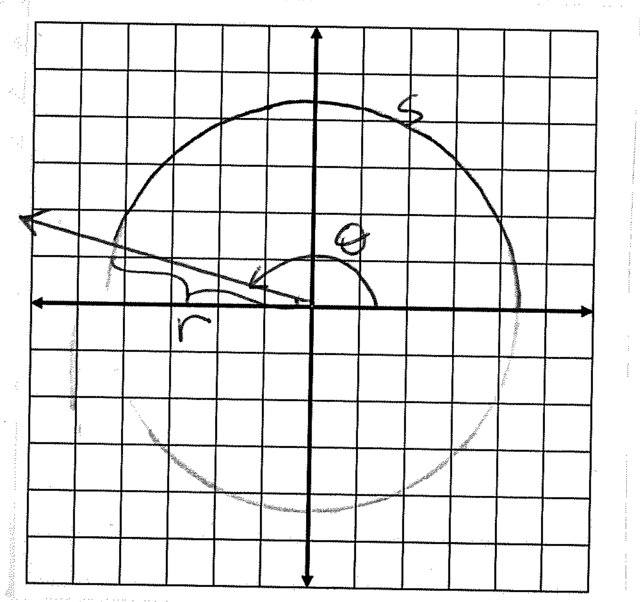
Note: An angle is in standard position if the initial side is on the positive horizontal axis and the vertex is at the origin.

Ex 6 Find the radian measure of θ.

(a) the length of s associated w/ θ is $\frac{\pi}{2}$ on a circle of radius 9.

(b) θ is the angle generated by $\frac{4}{5}$ of a clockwise rotation.

radian measure of an angle θ is the signed arc length s divided by the radius of the circle, i.e. $\boxed{\theta = \frac{s}{r}}$



Ex 7 Find the arc length of the arc associated w/ $\theta = \pi$ in a circle of radius 10.