Appendix B (Sequences & Series)

<u>sequence</u>: an ordered list/set of numbers that follow a specified pattern

<u>term</u>: each number in a sequence is called a term.

notation: $a_1, a_2, a_3, \ldots, a_n, \ldots$

or $\{a_n\}_{n=1}^{\infty}$ or $\{a_n\}$

(subscript on a is the counter (or index value) value, telling you which term of the sequence it is)

<u>Ex 1</u> List the first five terms of these sequences.

(a) $a_n = 2n-1$, $n=1,2,3,\ldots$

(b) $a_n = a_{n-1} + 5$, $a_1 = 7$
$n=2,3,\ldots$

<u>Recursive</u> (implicit formula): (recurrence)

a_n depends on previous terms ($a_n = f(a_{n-1})$)

<u>ex</u> $a_n = 2a_{n-1} + 1$, $a_1 = 4$

<u>Iterative</u> (explicit) formula: (direct)

a_n formula only depends on n (the formula is independent of previous terms) ($a_n = f(n)$)

<u>ex</u> $a_n = n^2$, $n=1,2,\ldots$

127

M1080

Appendix B (cont)

Ex 2 For each given sequence, (i) find next two terms, (ii) find recursive (recurrence) formula, and (iii) find iterative (explicit) formula for a_n.

(a) $\left\{ 2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \dots \right\}$

(b) $\{ 4, 7, 10, 13, \dots \}$

Appendix B (cont)

Limit of a Sequence

We say $\lim_{n\to\infty} a_n = L$ if sequence $\{a_n\}$ approaches L as n continues to increase. Then "sequence a_n converges to L." If the terms of $\{a_n\}$ do not approach a single, finite number L, then "sequence $\{a_n\}$ diverges."

Ex 3 Determine if you think these sequences converge or diverge. If they converge, what is L?

(a) $\left\{ \dfrac{(-1)^n}{2n^2} \right\}_{n=1}^{\infty}$

(b) $\left\{ \sin\left(\dfrac{n\pi}{2}\right) \right\}_{n=1}^{\infty}$

Appendix B (cont)

Infinite Series $a_1 + a_2 + \ldots + a_n + \ldots = \sum_{k=1}^{\infty} a_k$

the sum of an infinite sequence

Partial Sum

$S_n = \sum_{k=1}^{n} a_k$ is the n^{th} partial sum of the infinite series and $S = \sum_{k=1}^{\infty} a_k = \lim_{n\to\infty} S_n$.

(this may or may not be finite)

Ex 4 See if you can find the value of 0.9999....

start w/ $0.9999\ldots = 0.9 + 0.09 + 0.009 + \ldots = S$

$= \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \ldots$

(a) Find S_1, S_2, S_3, S_4.

(b) What is your guess for S?

(c) Let $n = 0.\overline{9}$. Finish your proof w/ some algebra.

Appendix B (cont)

Ex5 (a) Convince yourself that $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$.

(b) Then $\sum_{k=1}^{\infty} \frac{1}{k(k+1)} = \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1}\right) = S$

Find S_1, S_2, S_3, and S_n (the partial sums).

(c) Can you conclude anything about S?

Appendix B (cont)

Ex6 Suppose I get a large pizza and I want to split it among 3 friends. I cut the pizza into 4 equal-size pieces, giving one piece to each friend. Then I cut the leftover piece into 4 equal-size pieces, giving one piece to each friend again. I continue this process forever.

(a) Write the series that represents how much pizza each friend gets.

(b) What is the sum of that series?

M1080