$\boxed{M1220}$ 9.8 # 25

$f(x) = \sum_{n=0}^{\infty} a_n x^n$ is even fn, i.e. $f(-x) = f(x)$
$\forall\ x \in (-R, R)$

Prove $a_n = 0$ if n is odd

Pf Since $f(x) = f(-x)$, then

$$\sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} a_n (-x)^n$$

$$\Leftrightarrow \sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} a_n (-1)^n x^n$$

$\Leftrightarrow a_0 = a_0,\quad a_1 = -a_1,\quad a_2 = -a_2,\quad a_3 = -a_3,\quad a_4 = -a_4,$ etc.

i.e. $a_n = a_n$ for n even

and $a_n = -a_n$ for n odd

but the only way $a_n = -a_n$ is if

$$2a_n = 0$$

$$\Leftrightarrow a_n = 0$$

$\Rightarrow$ if n is odd, $a_n = 0$ must be true //

9.8 #27

$$\sin^{-1}x = \int_0^x (1-t^2)^{-1/2}\,dt$$

Find 1st four nonzero terms in Maclaurin series.

$$(1-t^2)^{-1/2} = 1 + \binom{-1/2}{1}(-t^2) + \binom{-1/2}{2}(-t^2)^2 + \binom{-1/2}{3}(-t^2)^3$$
$$+ \binom{-1/2}{4}(-t^2)^4 + \cdots$$

$$\binom{-1/2}{1} = \frac{-1/2}{1!} = -1/2$$

$$\binom{-1/2}{2} = \frac{-1/2(-3/2)}{2!} = \frac{3}{4(2)} = \frac{3}{8}$$

$$\binom{-1/2}{3} = \frac{-1/2(-3/2)(-5/2)}{3!} = \frac{-1\cancel{8}5}{8(\cancel{6})_{2}} = \frac{-5}{16}$$

$$\binom{-1/2}{4} = \frac{-1/2(-3/2)(-5/2)(-7/2)}{4!} = \frac{-5}{16}\left(\frac{-7}{2\cdot 4!}\right) = \frac{35}{768}$$

$$\Rightarrow (1-t^2)^{-1/2} = 1 + \frac{-1}{2}(-t^2) + \frac{3}{8}(t^4) + \frac{-5}{16}(-t^6) + \frac{35}{768}(t^8)$$
$$+ \cdots$$

$$= 1 - \frac{1}{2}t^2 + \frac{3}{8}t^4 + \frac{5}{16}t^6 + \cdots$$

$$\Rightarrow \sin^{-1}x = \int_0^x \left(1 - \frac{1}{2}t^2 + \frac{3}{8}t^4 + \frac{5}{16}t^6 + \cdots\right)dt$$
$$= \left(t - \frac{1}{6}t^3 + \frac{3}{40}t^5 + \frac{5}{112}t^7 + \cdots\right)\Big|_0^x$$
$$= x - \frac{1}{6}x^3 + \frac{3}{40}x^5 + \frac{5}{112}x^7 + \cdots$$