Math1220 Extra Final Review

This is basically an old final exam from a previous year. It will be good review for you to help you prepare for your final exam.

Part 1: Answer each of the questions 1 through 17.

1. Find the equation of the tangent line to the graph of

$$y=(1-2x^3)^\pi+\pi^x+\sinh(\tan^{-1}(x)) \quad \text{at } x = 0.$$

2. Find $\frac{dy}{dx}$. **Do NOT simplify your answer for this question. (Just get the derivative by itself.)**

$$y=(\ln(\cos^2(4x+3)))^{2x-1}$$

3. Find the convergence set for $\sum_{n=1}^{\infty}\frac{(-5)^n(x+1)^n}{2n+3}$.

4. Find the limit, if it exists. $\lim_{x\to 1^-}\left(\ln(1-x)\tan(\pi x)\right)$

5. Determine if this series converges absolutely, converges conditionally, or diverges. Show all your work, state which tests you used, and explain your reasoning.

$$\sum_{n=3}^{\infty}\left(1-\left(\frac{n-2}{n}\right)^n\right)$$

6. Determine if this series converges absolutely, converges conditionally, or diverges. Show all your work, state which tests you used, and explain your reasoning.

$$\sum_{n=2}^{\infty}\frac{4n^2+n-1}{\sqrt[3]{n^3+n^{10}+n^5-2}}$$

7. Determine if this series converges absolutely, converges conditionally, or diverges. Show all your work, state which tests you used, and explain your reasoning.

$$\sum_{n=1}^{\infty}\frac{(n+1)^3(-2)^{n+1}}{(2n-1)!}$$

8. For $f(x)=\frac{3x^3}{(1-2x)^3}$, find a power series to represent $f(x)$ and state its radius of convergence.

9. For $f(x)=\frac{5}{3-x}+1$

(a) Find the Taylor polynomial of order 3 centered about a = 2.

(b) Approximate f(2.1) using the polynomial from (a).

(c) Find a bound for the error in your approximation.

10. For Cartesian coordinates $(-4, 4\sqrt{3})$, find three different ways to represent this point in polar coordinates. (At least one of the points should have a negative r value.)

11. Find the slope of the tangent line to the graph of $r = 2 - \sin(2\theta)$ at

$$\theta = \frac{\pi}{3} \quad .$$

12. For the functions $r = 4\cos\theta$ and $r = 1 + \cos\theta$

(a) Graph the functions on the same coordinate axes and find the points of intersection (in polar coordinates).

(b) Set up (but do NOT evaluate) the area integral to find the area inside $r = 4\cos\theta$ and outside $r = 1 + \cos\theta$.

13. For the sequence given by $10^{\left(\frac{-5}{11}\right)}, 10^{\left(\frac{-11}{17}\right)}, 10^{\left(\frac{-17}{23}\right)}, 10^{\left(\frac{-23}{29}\right)}, \ldots$

(a) Find the expression representing a_n and tell what n-value your sequence begins with.

(b) Determine whether $\{a_n\}$ converges or diverges. If it converges, find $\lim_{n\to\infty} a_n$.

14. For $f(x) = \sinh(4x) + 5x^3 - 2^{-x} - 45$,

(a) Prove that $f(x)$ has an inverse, $f^{-1}(x)$.

(b) Evaluate $f^{-1}(-46)$.

(c) Evaluate $(f^{-1})'(-46)$.

15. Three people want to divide a cake. They cut it into four equal pieces with each person getting one piece. Then, they cut the leftover piece into four equal pieces, giving one piece to each person. They continue in this manner indefinitely. How much cake does each person get? (Show your work for all steps and show your reasoning using calculus.)

16. Find the limit, if it exists. $\lim_{x\to 0} \left(\sin(2x) + 3^{\frac{x}{5}}\right)^{\frac{5}{x}}$

Part 2: Choose 4 of the following 7 integrals to evaluate. (Leave your answers in **<u>exact</u>** form, no calculator necessary.) <u>You</u> must choose which problems to grade!

A. Grade: Yes or No (circle one)

$$\int \frac{x^4 + 4x^3 + 4}{x^3 - 4x^2}\, dx$$

B. Grade: Yes or No (circle one)

$$\int_0^1 \frac{10x}{(2x^2+2)\ \ln(x^2+1)}\, dx$$

C. Grade: Yes or No (circle one)

$$\int_0^6 \frac{2}{\sqrt[3]{(4-2x)^2}}\, dx$$

D. Grade: Yes or No (circle one)

$$\int \frac{3x^2}{\sqrt{25-x^2}}\,dx$$

E. Grade: Yes or No (circle one)

$$\int (5x+2)\cos(2x)\,dx$$

F. Grade: Yes or No (circle one)

$$\int_3^{\infty} \frac{(x-2)^5}{1+(x-2)^{12}}\,dx$$

G. Grade: Yes or No (circle one)

$$\int \frac{3x}{(\sqrt{4-x})^5}\,dx$$

Answers:

1. $y=(1+\ln\pi)x+2$
2. $\left(\ln\left(\cos^2(4x+3)\right)\right)^{2x-1}\left(2\ln\left(\ln\left(\cos^2(4x+3)\right)\right)-\dfrac{8(2x-1)\sin(4x+3)}{\cos(4x+3)\ln\left(\cos^2(4x+3)\right)}\right)$
3. $-\frac{6}{5}<x\leq-\frac{4}{5}$
4. 0
5. diverges, by nth term test
6. converges absolutely, because it's an all-positive series and passes LCT
7. converges absolutely by ART
8. $\sum_{n=0}^{\infty} 3(2^{n-1})(n+2)(n+1)x^{n+3}$, R = ½
9. (a) $f(x)\approx 6+5(x-2)+5(x-2)^2+5(x-2)^3$, (b) 6.555, (c) $\frac{50}{9^5}$
10. $(8,\frac{2\pi}{3}),(-8,\frac{-\pi}{3}),(8,\frac{-4\pi}{3})$
11. $\frac{4+\sqrt{3}}{5-4\sqrt{3}}$
12. (a) $(\frac{4}{3},\arccos\left(\frac{1}{3}\right))$, $(\frac{4}{3},-\arccos\left(\frac{1}{3}\right))$ (b) $A=2[\frac{1}{2}\int_0^{\arccos\left(\frac{1}{3}\right)}\left((4\cos\theta)^2-(1+\cos\theta)^2\right)d\theta]$
13. (a) $a_n=10^{-\left(\frac{6n-1}{6n+5}\right)}$ for n starting at 1; (b) $\frac{1}{10}$
14. (a) the derivative is always positive ==> f(x) is monotonically increasing ==> f-inverse exists; (b) 0; (c) $\frac{1}{4+\ln 2}$
15. 1/3 (set up as a convergent geometric series)
16. $3e^{10}$

A. $\frac{1}{2}x^2+8x-\frac{1}{4}\ln|x|+\frac{1}{x}+\frac{129}{4}\ln|x-4|+C$

B. diverges

C. $3\sqrt[3]{4}-6$

D. $\frac{75}{2}\arcsin\left(\frac{x}{5}\right)-\frac{3}{2}x\sqrt{25-x^2}+C$

E. $\frac{1}{2}(5x+2)\sin(2x)+\frac{5}{4}\cos(2x)+C$

F. $\frac{\pi}{24}$

G. $\frac{8}{\sqrt{(4-x)^3}}-\frac{6}{\sqrt{4-x}}+C$