Math1220 Midterm 1 Review Problems
Answer Key

1. $y=-4x+4+\frac{\pi}{2}$

2. $f^{-1}(x)=\frac{1+5\sqrt[3]{x}}{2-2\sqrt[3]{x}}$

3.
   (a) $y'=\frac{2\cos(3x)(-\sin(3x))(3)}{\cos^2(3x)}+\frac{3}{\sqrt{1-(3x-2)^2}}$

   (b) $y'=(5x+3)^{2x^2}\left(4x\ln(5x+3)+\frac{5(2x^2)}{5x+3}\right)$

   (c) $y'=\pi(1+x^4)^{\pi-1}(4x^3)+\pi^{1+x^4}(\ln\pi)(4x^3)$

   (d) $y'=-sech(\cos(2x))\tanh(\cos(2x))(-\sin(2x))(2)$

   (e) $y'=\frac{3}{3x-2}-12x^{-7}+12x^2-5\cos(5x)$

   (f) $y'=e^{\frac{1}{3x}}\left(\frac{-1}{3x^2}\right)+\frac{1}{e^{3x}}(-3)$

   (g) $y'=(x^3-1)^{\ln x}\left(\frac{1}{x}\ln(x^3-1)+\frac{3x^2(\ln x)}{x^3-1}\right)$

   (h) $y'=\frac{-\sin x}{\sqrt{(\cos x+3)^2-1}}$

4. $t=\frac{-10\ln(0.08)}{\ln 2}$ years

5. Evaluate each integral.
   (a) $5\ln|2x^2+x-7|+C$
   (b) $-5\arctan(\ln x)+C$
   (c) $\frac{4^{-1}-4^{-5}}{\ln 16}=\frac{1-4^{-4}}{4\ln 16}$
   (d) $\frac{-1}{\ln 2}\left(2^{\sqrt{3}/2}-2\right)$
   (e) $\frac{-1}{3}\ln 4$
   (f) $\frac{6^4-1}{\ln 36}$
   (g) $\frac{1}{2}\ln(e^{2x}+5)+C$
   (h) $\frac{5}{3}(\arcsin(x^3))+C$
   (i) $\frac{1}{4}\arctan\left(\frac{x^2}{2}\right)+C$

(j) $\frac{1}{4}\ln(x^4+4)+C$

6. $\frac{1}{14}$

7. $f'(x)=\frac{\sin x+1}{\cos^2 x}$ and since $\sin x$ is always between -1 and 1, then $1+\sin x$ must be between 0 and 2 (inclusive) which is always nonnegative. The denominator is also always positive wherever the function actually exists. This means that the derivative is always nonnegative in the restricted domain of the function. This is the same as saying that it's monotonic. (We can restrict the domain to be $(\frac{-\pi}{2}, \frac{\pi}{2})$.)

8. $\frac{1}{4}$

9. $f'(x)=-\left(\frac{2}{1+4x^2}+15(x-1)^2\right)$ which is always positive inside the parentheses since all coefficients are positive and the powers on x are even. Thus, the derivative is always negative which means the inverse function exists. $(f^{-1})'(11)=\frac{1}{f'(0)}=\frac{-1}{17}$

10.

(a) $\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{5x}=e^{15}$

(b) $\lim_{x\to\infty}(1)^{5x}=1$