

Math5700
Homework Set #5

Do these problems from the book:

3.3.1 #8, 9;

4.3.1 #2, 4;

4.3.3 #13;

4.3.4 #1, 5, 6, 10, 11, 14

And, then also do these induction proofs.

A. For the Fibonacci sequence, defined recursively as

$a_1 = 1$, $a_2 = 1$, $a_n = a_{n-1} + a_{n-2}$, $n \geq 2$, I claim the direct formula is

$$a_n = \frac{(1 + \sqrt{5})^n - (1 - \sqrt{5})^n}{\sqrt{5} \cdot 2^n} \text{ for all } n = 1, 2, 3, \dots$$

Prove this.

B. Prove that for all natural numbers n , $n^2 - n$ is even.

C. Make a conjecture about the sum $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{n}{(n+1)!}$ and prove your claim.

D. For f given recursively by $f(0) = 0$, $f(n) = f(n-1) + 3n + 2$ for all $n = 1, 2, \dots$ find an explicit formula for $f(n)$ and prove your formula is valid.