Math5700 Notes
Section 5.2
Divisibility Properties of Integers

Eulidean Algorithm for GCF: (one you can teach to younger kids)

1. GCF(180, 504)

2. GCF(180, 504)

Venn Diagram for LCM and GCF:

Claim: GCF(a,b)*LCM(a,b) = ab for all natural numbers a and b.

Proof:

Note: If a and b are relatively prime, then in their Venn diagram, we can draw their circles as mutually exclusive.

Diophantine Equations: equations with integer coefficients where we're only interested in integer solutions.

Example (stamp problem):
(a) Can the exact 93-cent postage for a package be put on the package using only 15-cent and 6-cent stamps?
(b) What if we need $1.93 postage on the package?

Claim: The postage, for this problem, needs to be ____ in order to create that amount using only 15-cent and 6-cent stamps.

Why?

Theorem 5.6:
For all natural numbers a and b, such that $a^2+b^2\neq 0$, the Diophantine equation $ax+by=GCF(a,b)$ has solutions.

Claim: For all natural numbers a and b, GCF(a,b)=1 iff there is an x and $y\in\mathbb{Z}$ such that $ax+by=1$.

Proof:

Now we claim that any two consecutive positive integers are always relatively prime. Why?

Theorem 5.8:
Let $a, b, c \in \mathbb{Z}$ and $d = GCF(a, b)$. Then $ax + by = c$ has solutions iff d is a factor of c.

And, if (x_0, y_0) is one solution of this equation, then the set of all integer solutions is given by
$$\left(x_0 - \frac{mb}{d}, y_0 + \frac{ma}{d}\right) \quad \forall m \in \mathbb{Z} \quad .$$

(see proof in book, section 5.2.3)

Pythagorean Triples
(another Diophantine equation)
$$a^2 + b^2 = c^2 \text{ such that } a, b, c \in \mathbb{N}$$
Can we find solutions?

We only need to find "primitive" triples, since if (a, b, c) is a solution, then so is (ma, mb, mc) for some positive m-value.

Process:

1. If we are only looking for primitive triples, then a must be odd and b must be even (or vice versa) because

2. c is odd because

3. (c-b) and (c+b) have no common factors.

Theorem 5.3:
If the product of two relatively prime integers u and v is a perfect square of an integer, then u and v are also perfect squares.

Proof:

4. Pythagorean Triples Formula

Other Bases

Theorem 5.15:
For every integer b, $b > 1$, every positive integer n has a unique representation in base b.

Example:
(a) Find base 4 representation of 365.

(b) Rewrite 365 in base 2.

(c) Rewrite 1000 in base 2 (binary), in base 8 (octal), and in base 16 (hexadecimal).

Example:

Given $x = 647_8$ and $y = 251_8$, find

x-y x+y xy

Claim: If the sum of the digits of a base 10 number is divisible by 9, then so is the number.

Proof: