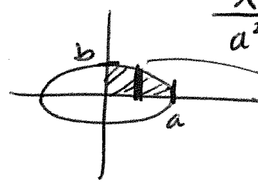
$\boxed{\text{HW \#6}}$

4) area of ellipse

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

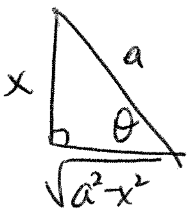


$$y^2 = b^2\left(1 - \frac{x^2}{a^2}\right)$$

$$y = b\sqrt{1 - \frac{x^2}{a^2}}$$

$$A = 4\int_0^a b\sqrt{1 - \frac{x^2}{a^2}}\,dx$$

$$= \frac{4b}{a}\int_0^a \sqrt{a^2 - x^2}\,dx$$



$$\frac{x}{a} = \sin\theta \qquad \Rightarrow \frac{\sqrt{a^2 - x^2}}{a} = \cos\theta$$

$$x = a\sin\theta$$

$$dx = a\cos\theta\,d\theta$$

$$\Rightarrow A = \frac{4b}{a}\int_0^{\pi/2} a\cos\theta\,(a\cos\theta)\,d\theta$$

$$= 4ab\int_0^{\pi/2} \cos^2\theta\,d\theta$$

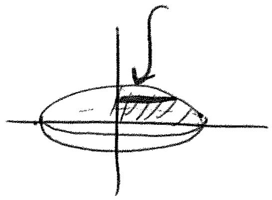
$$= \frac{4ab}{2}\int_0^{\pi/2} (1 + \cos(2\theta))\,d\theta$$

$$= 2ab\left(\theta + \frac{1}{2}\sin(2\theta)\right)\Big|_0^{\pi/2}$$

$$= 2ab\left[\left(\frac{\pi}{2} - 0\right) + \frac{1}{2}(\sin\pi \cancel{-} \sin 0)\right] \quad 0$$

$$= 2ab\left(\frac{\pi}{2}\right) = \boxed{ab\pi}$$

4) (a)



$$V = \pi \int_{-b}^{b} x^2 \, dy$$

but $x^2 = a^2\left(1 - \frac{y^2}{b^2}\right)$

$$\Rightarrow V = \pi \int_{-b}^{b} a^2\left(1 - \frac{y^2}{b^2}\right) dy$$

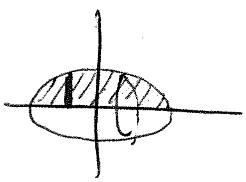
$$= \pi a^2 \left(y - \frac{1}{3b^2} y^3\right)\Big|_{-b}^{b}$$

$$= \pi a^2 \left((b - -b) - \frac{1}{3b^2}(b^3 + b^3) \right)$$

$$= \pi a^2 \left(2b - \frac{2}{3} b\right)$$

$$= \boxed{\frac{4\pi a^2 b}{3}}$$

4(b)



$$V = \pi \int_{-a}^{a} y^2 \, dx$$

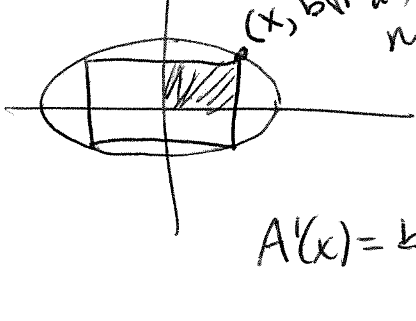
$$= \pi \int_{-a}^{a} b^2\left(1 - \frac{x^2}{a^2}\right) dx$$

$$= \pi b^2 \left(x - \frac{x^3}{3a^2}\right)\Big|_{-a}^{a}$$

$$= \pi b^2 \left(2a - \frac{2}{3} a\right) = \boxed{\frac{4\pi b^2 a}{3}}$$

4(c)

$\left(x, b\sqrt{1 - \frac{x^2}{a^2}}\right)$



maximize Q1 area

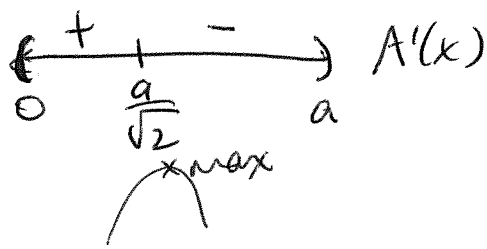
$$A = x\left(b\sqrt{1 - \frac{x^2}{a^2}}\right)$$

$$A'(x) = b\sqrt{1 - \frac{x^2}{a^2}} + xb\left(\frac{-2x/a^2}{2\sqrt{1 - x^2/a^2}}\right)$$

$$= \frac{b(1 - x^2/a^2) - b/a^2 (x^2)}{\sqrt{1 - x^2/a^2}} = \frac{b - \frac{2b}{a^2}x^2}{\sqrt{1 - x^2/a^2}} = 0$$

$$\Rightarrow x^2 = \frac{a^2}{2} \Rightarrow x = \frac{a}{\sqrt{2}} \text{ (in Q1)}$$

4) (c) (cont)

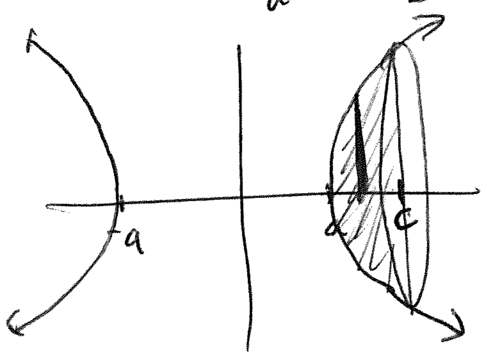


$\Rightarrow$ dimensions of entire rectangle (2x by 2y)

$\sqrt{2}a$ by $\sqrt{2}b$

4) (d) $b^2x^2 - a^2y^2 = a^2b^2$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



$c = \sqrt{a^2+b^2}$

$$V = \pi \int_0^{\sqrt{a^2+b^2}} y^2 \, dx$$

but $\frac{y^2}{b^2} = \frac{x^2}{a^2} - 1 \Rightarrow y^2 = b^2\left(\frac{x^2}{a^2} - 1\right)$

$$\Rightarrow V = \pi \int_a^{\sqrt{a^2+b^2}} b^2\left(\frac{x^2}{a^2} - 1\right) dx$$

$$= \pi b^2 \left(\frac{x^3}{3a^2} - x\right)\Big|_a^{\sqrt{a^2+b^2}}$$

$$= \pi b^2 \left[\left(\frac{\sqrt{(a^2+b^2)^3}}{3a^2} - \sqrt{a^2+b^2}\right) - \left(\frac{1}{3}a - a\right)\right]$$

$$= \pi b^2 \left[\frac{\sqrt{(a^2+b^2)^3}}{3a^2} - \sqrt{a^2+b^2} + \frac{2}{3}a\right]$$