5.1 Arithmetic and Geometric Sequences

Ex 1 Classify each sequence as arithmetic, geometric or neither.

(a) $2, -6, 18, -54, \ldots$

(b) $11, 8, 5, 2, \ldots$

(c) $1, 4, 9, 16, \ldots$

Ex 2 Find formula for a_n given:

(a) $a_1 = 3$, $d = 5$, arithmetic sequence

(b) $a_1 = 4$, $d = \frac{1}{2}$, geometric sequence

Arithmetic Sequence

common difference $= d$

$a_n = a_{n-1} + d$

given a_1

or $a_n = a_1 + (n-1)d$, $n = 2, 3, 4, \ldots$

Geometric Sequence

common ratio $= d$

$a_n = d a_{n-1}$, given a_1

or $a_n = a_1(d^{n-1})$, $n = 2, 3, 4, \ldots$

Arithmetic Sequence Sum

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Geometric Sequence Sum

$$S_n = \frac{a_1(1-d^n)}{1-d}$$

5.1 (cont)

Ex 3 Given $a_1 = 5$, $a_{99} = 54$ and $\{a_n\}$ is an arithmetic sequence, find the 50th term (a_{50}) of the sequence.

Ex 4 Given $a_1 = \frac{1}{16}$, $a_7 = 256$ and $\{a_n\}$ is geometric sequence, find a_{20}.

Ex 5 Find the sum of the first 100 terms of these sequences.

(a) 1, 10, 19, 28, ...

(b) 1, −5, 25, −125, ...

5.2 Simple and Compound Interest

Ex1 Jose deposits $4,000 into an account earning 4.2% annual interest compounded monthly and leaves it there for 5 years. At the end of those 5 years, what is his account balance?

Simple Interest

$$S = P(1+rt)$$

P = principal
t = # years $ in acct
r = annual interest rate (as decimal)
S = acct balance after t yrs

Compound Interest

$$S = P\left(1+\frac{r}{n}\right)^{nt}$$

n = # compoundings per year

Continuous Compound Interest

$$S = Pe^{rt}$$

Ex2 You have the choice between these accounts. Which is best?

[A] r = 5.5% quarterly compounding

[B] r = 5.25% monthly compounding

APY

① $APY = \left(1+\frac{r}{n}\right)^n - 1$ periodic compounding

② $APY = e^r - 1$ continuous compounding

5.2 (cont)

Ex 3 Kade just discovered that their grandmother left them $15,000 to help pay for college. But they're only 10 years old. Because Kade is a wise investor, they put the money into an account earning 5.4% annual interest compounded monthly for 8 years until they start college. (a) At that time, how much will their account be worth? (b) How much interest did they earn in that time?

Ex 4 How long will it take to double your investment

(a) in an account with
$r = 7.2\%$
$n = 12$

(b) in a compound continuously account with $r = 6\%$

5.3 Future Value (FV) of Annuities

Ex1 Liping wants to save for her retirement. She saves $400 per month for 30 years. She puts it all in an account that earns 5.1% interest compounded monthly. How much will her retirement account be worth at the end of the 30 years? (Assume she deposits her monthly payments at the end of each month.)

FV of ordinary Annuity

(payments at end of each period)

$$S = R\left(\frac{(1+r_c)^N - 1}{r_c}\right)$$

$r_c = \frac{r}{n}$, R = regular payment

S = future value

N = total # of payments

$N = nt$

FV of Annuity Due

(payments at beginning of each period)

$$S_{due} = R(1+r_c)\left(\frac{(1+r_c)^N - 1}{r_c}\right)$$

Sinking Fund Payment

$$R = S\left(\frac{r_c}{(1+r_c)^N - 1}\right)$$

5.3 (cont)

Ex 2 Omar would like to know how much he needs to deposit in an investment account every month in order to have $12,000 for his dream vacation in 5 years. He finds a safe account that pays 3.8% annual interest, compounded monthly. He makes his payments at the beginning of each month.

Ex 3 Barry wants $2,000,000 to retire. He makes quarterly payments to an account earning 6% annual interest, compounded quarterly. Each payment is $2000. How many payments must he make to reach his goal? How many years is that?

5.3 (cont)

Ex 4 Bailey's grandmother died and left Bailey $10,000. After consulting with a financial advisor, Bailey decides she will invest the money in an account earning 4.5% annual interest, compounded monthly. In addition, she will continue depositing $350 per month into the same account for 27 years. How much will her account be worth after those 27 years?

5.4 Present Value (PV) of Annuities

Ex) You are trying to decide when you can retire. You estimate you'll need to withdraw $2500 at the end of each month for living expenses, for 35 years. The account earns 5.9% annual interest, compounded monthly. How much must be in the account when you retire?

PV of Ordinary Annuity

$$P = \frac{R\left(1-(1+r_c)^{-N}\right)}{r_c}$$

R = withdrawal amt.

$r_c = \frac{r}{n}$, $N = nt$

P = PV

PV of Annuity Due

$$P_{due} = \frac{R(1+r_c)\left(1-(1+r_c)^{-N}\right)}{r_c}$$

PV of Deferred Annuity

$$P = \frac{R\left(1-(1+r_c)^{-N}\right)}{r_c(1+r_c)^{m}}$$

m = # compounding periods that withdrawals are deferred

5.4 (cont)

Ex 2 Daryl receives an inheritance of $60,000. He puts it into an account earning 3.6% interest compounded monthly, and he leaves it there for 35 years. At that point in time, he wants to start withdrawing monthly payments, at the beginning of each month for another 25 years. How much will each of his monthly withdrawals be?

Ex 3 Ulysses receives a lottery prize of $560,000. He'll receive payments at the end of each quarter for 14 years of $10,000. If the money is in an account earning 5% interest compounded quarterly, what is the real value of the lottery prize today?

5.5 Amortization and Loans

Ex1 Debra's consulting business is borrowing $65,000 on a 4-year, 4.2% amortized term loan. Payments will be made monthly.

(a) What will the payment amount be?

(b) How much interest will she pay for this loan?

Periodic Payment of Amortized Loan

$$R = S\left(\frac{r_c}{1-(1+r_c)^{-N}}\right)$$

S = loan amt
R = payment amt
N = total # payments = nt

Total Interest Paid

$NR - S$
(total paid − loan amt)

Loan Payoff Amount

$$S_{N-k} = R\left(\frac{1-(1+r_c)^{-(N-k)}}{r_c}\right)$$

k = # payments already made

Ex 2 The Ramirez family is considering refinancing their mortgage for their home. They still owe $130,000 on their house loan and the best offer they found is a loan with annual interest rate of 3.1%, compounded monthly. If they want to take out a 15-year mortgage, what will their monthly payments be? Create an amortization schedule for this loan.

5.5 (cont)

Ex 3 Refer back to the Ramirez family in Ex 2. If they get a cash bonus at work at the end of year 6 and pay off the rest of the loan, how much money (from unpaid interest) will they save by not making the rest of the payments?