

Math1090 Midterm 2 Review
Sections 2.1-2.5, 3.1-3.3

1. Solve the system of linear equations using Gauss-Jordan elimination.

$$5x + 20y - 15z = -155$$

(a) $-2x - 7y + 13z = 85$

$$3x + 14y + 6z = -43$$

$$x + z = -2$$

(b) $x = -6$

$$y + z = 11$$

$$x - y + z = 5$$

(c) $x - y + 2z = 13$

$$-x + 3z = 27$$

2. Given the matrices

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 7 & -1 \end{bmatrix} \quad C = \begin{bmatrix} 4 & -3 \\ 6 & 5 \\ -7 & 10 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 5 & 8 & 13 & 21 \end{bmatrix},$$

calculate each of the following. If a computation is not possible, state that and give the reason why.

(a) A zero matrix the same size as D.

(b) C^T

(c) $11037B - 5918D$

(d) $4A - B$

(e) AB

(f) BA

3. Consider the following matrix, whose entries depend on a variable x

$$A(x) = \begin{bmatrix} 2-x & 1 \\ 1 & 1-x \end{bmatrix}.$$

Answer the following questions.

(a) The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is the quantity $ad - bc$. Find the determinant of $A(x)$. Your

final answer should be a quadratic function $f(x)$ in standard form.

(b) Let $f(x)$ be the quadratic function you found in part (a). Find the x -intercepts for $y = f(x)$. (Give them in exact form, not approximate values.)

(c) What is the vertex of the parabola?

(d) Graph the function $y = f(x)$ on a Cartesian coordinate plane. Label the vertex and x -intercepts.

4. A local company sells ski equipment and wants to maximize their profit. The demand function is given by: $p = 90 - x$ where p is the price of one unit in dollars and x is the number of units sold. The cost function is given by $C(x) = 30x + 500$, again in dollars.

(a) What is the revenue function? (Hint: Revenue is the price times the number of units sold at that price.)

(b) What is the profit function?

(c) How many units need to be produced and sold to break even?

(d) How many units must the company sell to maximize profit?

(e) What should the price of one unit be to maximize profit?

(f) What is the maximum profit?

5. Find the inverse matrix for the following matrices.

(a) $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 1 & 7 \\ 1 & 1 & 8 \\ 1 & 0 & 2 \end{bmatrix}$

6. A global company sells washing machines, and a regional manager wants to know the optimum number to have in stock for the stores she is responsible for. A mathematical model tells you that the supply function is given by $p = q^2 - 37q - 120$ and the demand function is given by $p = 780 + 45q - q^2$. What is the equilibrium point?

7. Given the function $y = f(x) = \frac{-1}{2}(x-1)^2 + 8$, answer the following questions.

- Find the vertex.
- Find the axis of symmetry.
- Is this parabola concave up or down?
- Find the x-intercept(s).
- Find the y-intercept.
- Sketch the graph.

8. If you're doing Gauss-Jordan elimination and you get this augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 3 & 4 & 3 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad \text{what is the solution to the original system of equation?}$$

9. If you're doing Gauss-Jordan elimination and you get this augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 3 & 4 & 3 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 1 \end{array} \right] \quad \text{what is the solution to the original system of equation?}$$

Note: The next problems are from Quiz 5, to practice again.

10. Given $A = \begin{bmatrix} 4 & 0 & -5 \\ 1 & 3 & -8 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 5 & 1 \\ 2 & 7 & -3 \\ 6 & 14 & 5 \end{bmatrix}$, find the inverse matrices A^{-1} , B^{-1} , and C^{-1} . If the computation is impossible, state that and the reason why it's not possible.

11. Follow these steps to solve the system of equations.

$$x - y = 5$$

$$3x + 2y = 25$$

- Write corresponding matrix equation, $AX = B$, i.e. write what A, X, and B are.
- Find A^{-1} .
- Use A^{-1} to solve the system of equations.

12. An investor has \$600,000 to invest. There are three ways to invest this money. Municipal bonds pay 8% interest every year. Blue chip stocks should return about 12% annually. Growth or speculative stocks should pay about 15% annually. The investor wants to get a 10% return overall on her investments. Also, she wishes to invest twice as much in blue chip stocks as in growth or speculative stocks. How much should she invest in each type of investment?

Answer Key:

1. (a) (1, -5, 4)
(b) (-6, 7, 4)
(c) (-3, 0, 8)

2. (a) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & 6 & -7 \\ -3 & 5 & 10 \end{bmatrix}$

(c) This computation is impossible because B and D have different sizes.

(d) $\begin{bmatrix} 7 & 4 \\ -3 & 5 \end{bmatrix}$

(e) $\begin{bmatrix} 9 & 1 \\ 8 & -1 \end{bmatrix}$

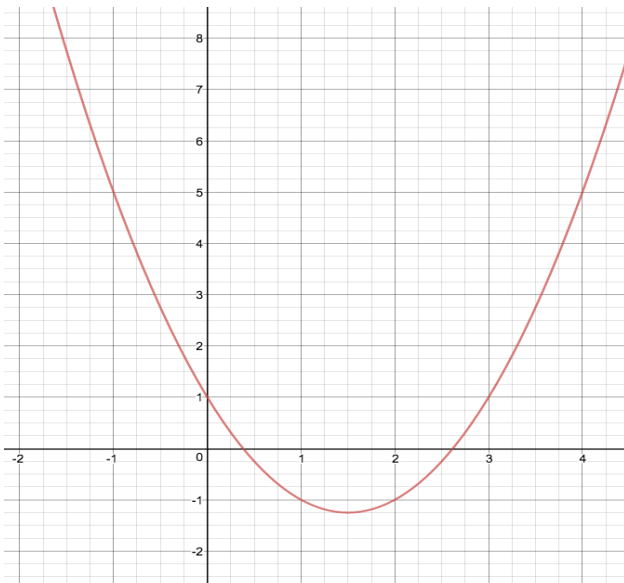
(f) $\begin{bmatrix} 2 & 1 \\ 13 & 6 \end{bmatrix}$

3. (a) $f(x) = x^2 - 3x + 1$

(b) $\left(\frac{3+\sqrt{5}}{2}, 0\right)$ and $\left(\frac{3-\sqrt{5}}{2}, 0\right)$

(c) $\left(\frac{3}{2}, \frac{-5}{4}\right)$

(d)



4. (a) $R(x) = 90x - x^2$

(b) $P(x) = -x^2 + 60x - 500$

(c) $x = 10$ or 50

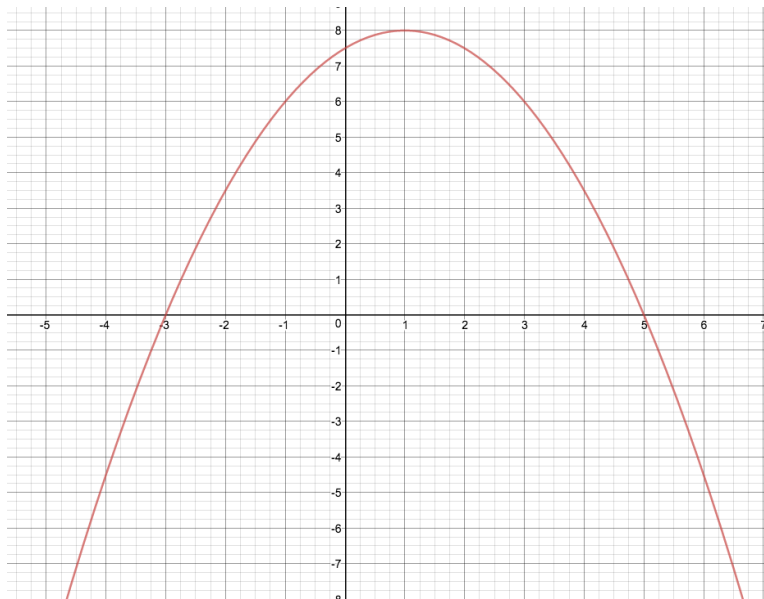
- (d) 30 units
- (e) \$60
- (f) \$400

5. (a) $A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

(b) $A^{-1} = \begin{bmatrix} 2 & -2 & 1 \\ 6 & -5 & -1 \\ -1 & 1 & 0 \end{bmatrix}$

6. (50, 530)

- 7. (a) (1, 8)
- (b) $x = 1$
- (c) concave down
- (d) (-3, 0) and (5, 0)
- (e) (0, 7.5)
- (f)



8. $(2t-21, 8-2t, t)$

9. N.S.

10. (i) We can't find A^{-1} because A is not a square matrix.

(ii) $B^{-1} = \begin{bmatrix} 1/5 & -1/10 \\ 2/5 & 3/10 \end{bmatrix}$ or $\begin{bmatrix} 0.2 & -0.1 \\ 0.4 & 0.3 \end{bmatrix}$

(iii) C^{-1} Does not exist (DNE); we get zeros in one row on the left of the augmented matrix which means this matrix has no inverse.

11. (a) $A = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 5 \\ 25 \end{bmatrix}$

(b) $A^{-1} = \begin{bmatrix} 2/5 & 1/5 \\ -3/5 & 1/5 \end{bmatrix}$

(c) $X = A^{-1}B = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$ so intersection point or solution is the point (7, 2)

12. bonds: \$360,000; blue chip stocks: \$160,000; growth stocks: \$80,000