11.1 Derivatives of Logarithmic Functions

Ex 1: Compute the following derivatives. (a) $D_x(3\ln(2x)+2x^4+7)$	Logarithm Defn: For $a>0$ and $a\neq 1$ and $b>0$, $\log_a b=c \Leftrightarrow a^c=b$
(b) $D_x(\ln(x^3+4)+x^5+e^2)$	Derivative of logarithm functions: $D_x(\ln x)=\frac{1}{x}$, $x>0$ and $D_x(\ln\lvert x\rvert)=\frac{1}{x}$, $x\neq 0$ *For* $a>0$ and $a\neq 1$, $D_x(\log_a x)=\frac{1}{x\ln a}$ Note: Remember that you can also use these formulas along with the chain, product and quotient rules for derivatives!!!
(c) $D_x(\ln(3\mathrm{x}-\sqrt{x^3+1}))$	Log Properties Reminder: $a, b\in\mathbb{R}^+$, $r\in\mathbb{Q}$ (1) $\ln 1=0$ (2) $\ln(ab)=\ln(a)+\ln(b)$ (3) $\ln\left(\frac{a}{b}\right)=\ln(a)-\ln(b)$ (4) $\ln(a^r)=r\ln(a)$ Also (5) $e^{\ln a}=a$ and (6) $\ln(e^a)=a$

Ex 2: Compute these derivatives. (a) $D_x\left(\dfrac{x^2+4}{9-\ln(3x^2-5)}\right)$	(b) $D_x\left((x^4+9x)\ln(5x+3)\right)$

Ex 3: My mom is really doing a great job with Hot Chippies! She's made changes to the marketing plan and recently she's determined that our current revenue function is given by $R(x)=\dfrac{1000x}{\ln(5x+10)}$.

(a) Find the marginal revenue function.

(b) Find the marginal revenue function when 50 cookies are sold. Interpret this result.

11.2 Derivatives of Exponential Functions

Ex 1: (a) Simplify this expression. $e^{\ln w^3 - 4\ln u}$	Remember: $e^{\ln x} = x \ , \ x > 0$ $\ln(e^x) = x \quad \forall\, x$
(b) Find the derivative $\frac{dy}{dx}$ for $y = (1+e^x)^3 + x^4$	Derivatives of Exponential Functions: $D_x(e^x) = e^x$ $D_x(a^x) = a^x(\ln a) \ , \ a > 0$
(c) Find the derivative $\frac{dy}{dx}$ for $y = \frac{e^{-2x}}{x^{2/3}}$	

11.2 (continued)

Ex 2: Find the following derivatives.

(a) $D_x\left(e^{x^3+2\ln(x+1)}\right)$	(b) $D_x\left(x^2 3^{x^2}\right)$

Ex 3: Find $\frac{dy}{dx}$ for $y=\ln(e^{-x}+5x)$

Ex 4: Find the equation of the tangent line to $y=(x+5)2^{-3x}$ at x = 0.

11.3 Implicit Differentiation

Setup:

Explicit Functions: We can write these sorts of functions as y = stuff in terms of x. examples: (a) $y=x^3+9x+8$ (b) $y=3^{2x}+x^3$ (c) $y=2\ln(x^3+4x)$	Implicit Functions: We canNOT write these sorts of functions/relations as y = stuff in terms of x. In other words, the y and x cannot be separated out so easily in the equation of two variables that relates them. examples: (a) $y^2+xy=x^3+9x+8$ (b) $y=3^{2xy}+x^3+y^3$ (c) $e^y+(xy)^2=2\ln(x^3+4x)$

Note: When we're using implicit differentiation, we apply the derivative operator to both sides of the equation and take the derivative with respect to x, remembering that y depends on x. We will still need to know the chain, product and quotient rules.

Ex 1: Find $\frac{dy}{dx}$ for each of these implicit functions.

(a) $x^2+2x^2y+3xy=0$
(b) $y^3=x^2\ln x$

Ex 2: Find y'' at the point (3, 4) if $x^2+y^2=25$.

Ex 3: Find $\frac{dy}{dx}$ for the curve given by $\frac{1}{x}+\frac{1}{y}=1$.

Ex 4: Find the equation of the tangent line for $x^2-y^3=2x$ at the point (1, -1).

11.4 Related Rates

Ex 1: A 20 ft ladder is leaning against a wall. The bottom of the ladder is sliding out from the wall at a rate of 0.5 feet per second. How fast in the top of the ladder sliding down the wall when the bottom of the ladder is 10 feet away from the wall?	Strategy for related rates problems: 0. First notice that the problem has two rates in it (one rate that is given and the other rate is unknown). 1. Write down all the information from the problem and draw a picture where that's relevant. 2. Find an equation that relates the two variables that are in the rates (derivatives). 2-1. If you have an equation with three variables, instead of the two variables you want, then you need to use some of the information give in the problem statement to get rid of one of those variables and have ONLY the two variables in the equation that are in the rates you have written down in step 1. 3. Differentiate the entire equation with respect to time, remembering that the variables you have depend on time. 4. Solve for the rate we want. Note: THIS is the time to plug in the particular values of the variables that the problem statement gave you.

11.4 (continued)

Ex 2: A spherical soap bubble retains its spherical shape as it changes in size. How fast is the radius increasing when the radius is 3 inches, if air is being blown into the bubble at a rate of 2 cubic inches per second?

11.4 (continued)

Ex 3: My mom is hard at work again, for Hot Chippies. Her arthritis is better right now and she'd like to increase the number of cookies she bakes while I'm teaching. She gave me this problem yesterday: Suppose our weekly revenue and cost functions are given by $R(x)=300x-0.001x^2$ and $C(x)=4000+30x$. She wants me to find out how fast the profit is changing when the number of cookies baked/sold is 50. She also told me that the number of cookies baked will increase at a rate of 5 cookies per day.

11.5 Applications in Business and Economics

<table>
<tr>
<td>Ex 1: Suppose the demand for a product is given by $(p+1)\sqrt{q+1}=1000$.
(a) Find the elasticity of demand when p = 39.</td>
<td><u>Elasticity of Demand</u>:

The elasticity of demand at the point (q_A, p_A) is
$\eta=-\frac{p}{q}\cdot\frac{dq}{dp}\Big|_{(q_A, p_A)}$

(η is called "eta"; it's a Greek letter)

If $\eta>1$, the demand is elastic, and the percent decrease in demand is greater than the corresponding percent increase in price.

If $\eta<1$, the demand is inelastic, and the percent decrease in demand is less than the corresponding percent increase in price.

If $\eta=1$, the demand is unitary elastic, and the percent decrease in demand is approximately equal to the corresponding percent increase in price.

<u>Elasticity and Revenue</u>:
Elastic $\eta>1$ means $\frac{dR}{dp}<0$ (So if price increases, revenue decreases, and if price decreases, revenue increases.)

Inelastic $\eta<1$ means $\frac{dR}{dp}>0$ (So if price increases, revenue increases, and if price decreases, revenue decreases.)

Unitary elastic $\eta=1$ means $\frac{dR}{dp}=0$ (So if price increases or decreases, there will be no change in revenue. This means Revenue is optimized at this point.)</td>
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<tr>
<td colspan="2">(b) What type of elasticity is this?</td>
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11.5 (continued)

Ex 2: If the demand for a product is given by $2p^2 q = 10{,}000 + 9{,}000p^2$, answer the following questions.

(a) What is the elasticity when price is \$50?

(b) Find the point where demand is unitary elastic.

Ex 3: If the demand function is given by $p=38-2q$ and the supply function before taxation is given by $p=8+3q$, what tax per item will maximize the total tax revenue?	Maximizing Total Tax Revenue: Strategy-- to find the tax per item (under pure competition) that will maximize total tax revenue. 1. Write the supply function after taxation. 2. Find equilibrium between demand and supply functions. In other words, solve for t in terms of q (where q = quantity demanded/supplied, p = price and t = tax) by using substitution. 3. Create the total tax revenue function given by T = tq. Then take the derivative $\frac{dT}{dq}$. 4. Set that derivative equal to zero, i.e. $\frac{dT}{dq}=0$, and solve for q. Verify that this gives you a max T value (not a min T value). 5. Substitute the value of q in the equation for t (from step 2). This is the value of the tax, t, that will maximize T, the total tax revenue.