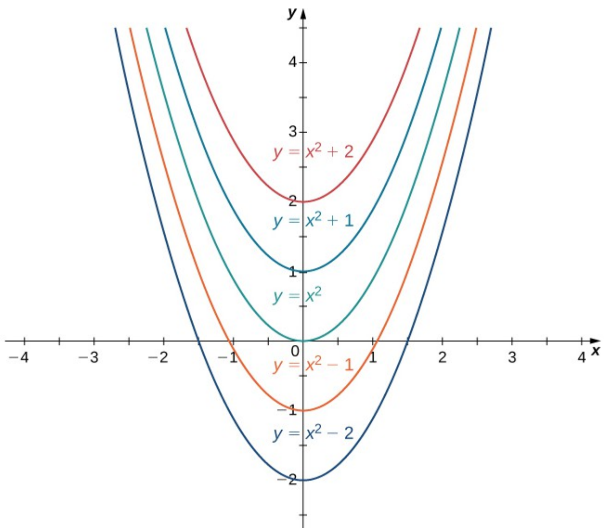
12.1 The Indefinite Integral

Why the "plus C"? 	<u>Antiderivative</u>: $\int f(x)\,dx = F(x)+C$ read "integral of f of x dx is F of x plus C". • f(x) is the integrand function • dx is the differential of the integral. <u>Antiderivative Rules</u>: Rules that "undo" derivatives. 1. $\int 1\,dx = x+C$ 2. $\int x^r\,dx = \frac{x^{r+1}}{r+1}+C$ (Power Rule) for $r \in \mathbb{Q}$, $r \neq -1$ 3. $\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$ 4. $\int e^x\,dx = e^x + C$ Note: Compare a couple derivatives with antiderivatives. $\frac{d}{dx}(x)=1 \Leftrightarrow \int 1\,dx = x+C$ $\frac{d}{dx}(\ln x)=\frac{1}{x}$, $x>0$, $\Leftrightarrow \int \frac{1}{x}\,dx = \ln x + C$
Ex 1: Evaluate the following antiderivative. $\int (x^3+4+\sqrt{x})\,dx$	Antiderivative (aka Indefinite Integral) operator is a linear operator. That is, it satisfies both of these properties. (a) $\int (f(x)+g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$ (i.e. antiderivatives distribute through addition) AND (b) $\int k\,f(x)\,dx = k\int f(x)\,dx$ for any constant *k* (i.e. antiderivatives commute with multiplication by a constant).

Ex 2: Evaluate these integrals. (a) $\int \left(\frac{2}{x^2}+x^{2/3}+\ln(5)\right)\ dx$	(b) $\int \frac{x(x+1)^2}{\sqrt{x}}dx$

Ex 3: I have finally had enough time to help my mom grow our cookie business, Hot Chippies. We've done some calculations recently and have discovered that our marginal revenue is approximated by $\overline{MR}(x)=300-0.2\,x$. Find the total revenue if our company were to sell 1000 cookies (over some period of time, obviously, we can't make 1000 cookies in one day in my home kitchen).
(Hint: At some point, you might have to ask yourself--"what is the revenue if we sell zero cookies?")

12.2 The Power Rule (aka u-substitution) & 12.3 Integrals Involving Exponential and Logarithmic Functions

Ex 1: If $u = \ln x + x^3$, find the differential du.	<u>Differentials:</u> If y = f(x) is a differentiable function with $\frac{dy}{dx} = f'(x)$, then the differential of y is dy and the differential of x is dx, and $dy = f'(x)dx$.
Ex 2: Evaluate the integral. $\int (4x^2 - 3\text{x})^5 (8x - 3)\ dx$	<u>Generalized Power Rule for Integration:</u> $\int (u(x))^n\ dx = \frac{(u(x))^{n+1}}{n+1} + C\ ,\ n \neq -1$ This is usually just called "u-sub" which is short for u-substitution. This technique of integration essentially undoes the chain rule for differentiation. Reminder-- <u>Antiderivative Rules</u>: Rules that "undo" derivatives. 1. $\int 1\, dx = x + C$ 2. $\int x^r\, dx = \frac{x^{r+1}}{r+1} + C$ (Power Rule) for $r \in \mathbb{Q}\ ,\ r \neq -1$ 3. $\int \frac{1}{x} dx = \ln\|x\| + C$ 4. $\int e^x\, dx = e^x + C$

Ex 3: Compute the integrals.

(a) $\int \frac{9y}{5y^2-3}\,dy$	(b) $\int 3x\,e^{6x^2-1}\,dx$
(c) $\int \frac{e^x}{e^x-5}\,dx$	$\int \left(9^{2x}-3^{4x}\right)dx$

12.2 & 12.3 (continued)

Ex 4: Compute these integrals.

(a) $\int e^{2x-1}\,dx$	(b) $\int (2x-1)^{e}\,dx$

Bonus Question: Evaluate these integrals, assuming $m \neq 0$, $a \neq -1$ and $n, m, b \in \mathbb{R}$.

(a) $\int e^{mx+b}\,dx$	(b) $\int (mx+b)^{n}\,dx$

Ex 5: Find the following integrals.

(a) $\int \frac{\ln(x^2)}{x}\, dx$	(b) $\int \frac{x^4-2x^2+x}{x^2-2}\, dx$ (Hint: Do long division.)

12.4 Applications of the Indefinite Integral in Business and Economics

Ex 1: A firm's marginal cost is given by $\overline{MC}(x)=3x+50$ and the total cost of producing 20 unites is \$2,000. What is the total cost function?

Ex 2: A firm's marginal cost is given by $\overline{MC}(x)=6x+60$ and the marginal revenue is $\overline{MR}(x)=180-2x$ and its total cost of production of 10 items is \$1000.

(a) Find the optimal level of production. (Hint: this happens when profit is maximized, which is when $\overline{MC}=\overline{MR}$.)

(b) Find the profit function.

(c) Find the profit (or loss) at the optimal production level.

Ex 3: If national consumption is \$9 billion when income is zero, and if the marginal propensity to consume is 0.30, answer the following questions. (a) What is the consumption function? (b) What is consumption when disposable income is \$20 billion?	National Consumption and Savings: If C represents the national consumption (in billions of dollars), then a **national consumption function** has the form C = f(y) where y is disposable national income (also in billions of dollars). The **marginal propensity to consume** is the derivative of the national consumption function with respect to y, i.e. $\frac{dC}{dy} = f'(y)$. Note: If we know the marginal propensity to consume, then we can take the integral and get back the national consumption function. If S represents the national savings, we can assume that the disposable national income is given by y = C + S (where C is the consumption function above). Then the **marginal propensity to save** is $\frac{dS}{dy} = 1 - \frac{dC}{dy} = 1 - f'(y)$.
Ex 4: Suppose that the marginal propensity to save is $\frac{dS}{dy} = 0.2 - \frac{1}{\sqrt{3y+7}}$ (in billions of dollars) and the consumption is \$6 billion when disposable income is \$0. Find the national consumption function.	