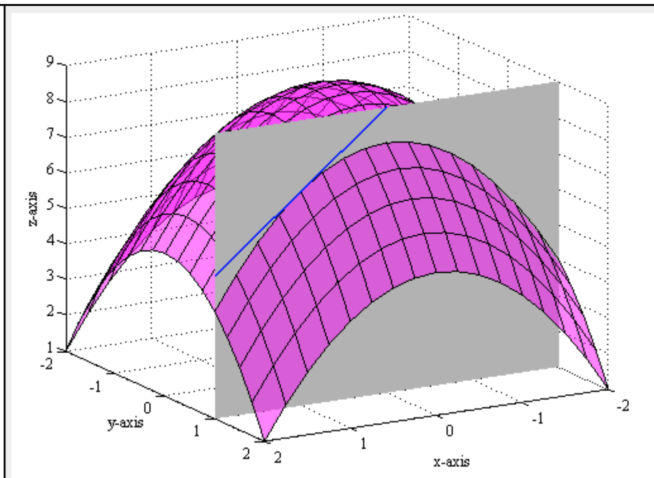
14.1 Functions of Two or More Variables

Ex 1: Let $f(x,y,z)=2\sqrt{x}+xyz^2+\ln(3y)$ (a) What is the domain of this function? (b) What dimension space does the graph of this function live in? (c) Find $f(1, \frac{1}{3}, -2)$.	Note: $y=f(x)$ is a function of one input variable and its graph lives in 2-d (one input + one output = 2-d) space $z=f(x,y)$ is a function of two input variables and its graph lives in 3-d (two inputs + one output = 3-d) space $w=f(x,y,z)$ is a function of three input variables and its graph lives in 4-d (three inputs + one output = 4-d) space
Ex 2: Find the domain of the function $f(x,y)=\frac{x^2+4}{\sqrt{y^2-z^2}}$	Domain still asks for the set of allowed variable values for the input variables.
Ex 3: The cost per day to society of an epidemic is $C(x,y)=20x+200y$ where C is in dollars, x is the number of people infected on a given day and y is the number of people who die on a given day. If 14,000 people are infected and 20 people die on a given day, what is the cost to society?	

14.2 Partial Derivatives

Ex 1: Find f_x and f_y given $f(x,y)=\ln(x^2-y^2)$.	(Note: wrt means "with respect to".) Given $z=f(x,y)$ the partial derivative of z wrt x is $f_x(x,y)=z_x=\frac{\partial z}{\partial x}=\frac{\partial f(x,y)}{\partial x}$. Likewise for the partial derivative of z wrt y. $f_y(x,y)=z_y=\frac{\partial z}{\partial y}=\frac{\partial f(x,y)}{\partial y}$ To find the partial derivative of f wrt x, for example, we pretend that y is a constant, and just do regular differentiation. We can do this because indeed as far as x is concerned, y is a constant, since they are independent input variables. Geometrically, when we find f_x at a particular point, we're getting back a number that represents the slope of the surface (in 3d) at that point, facing in the positive x direction. Likewise, f_y at a particular point would give back a number that tells us the slope of the surface at that point facing in the positive y direction.
Ex 2: Find the four second order partial derivatives for $f(x,y)=2x^3y-\ln y+e^x$.	For f_{xyy} , work "inside to outside" (or left to right) for this notation, i.e. find f_x then differentiate wrt y $(f_x)_y=f_{xy}$ and then differentiate again wrt y $(f_{xy})_y=f_{xyy}$. Notation: $f_{xyy}=\frac{\partial^3 f}{\partial^2 y\,\partial x}=\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)\right)$ In Leibniz notation, we read the partial derivative from right to left to enact it.

Ex 3: Imagine you are on the surface $z=x^4+4xy^2+e^{xy}$ at point $(1,-2)$. (a) Find the slope of the tangent to this surface in the positive x-direction.	 This shows a tangent line to a 3-d surface that's in the x-direction, i.e. a way to visualize the partial derivative wrt x.
(b) Find the slope of the tangent to this surface in the positive y-direction.	(c) Find the four partial second derivatives for this function.

Ex 4: Well, as you might suspect, my mom has made a go of the brownie business in addition to Hot Chippies. So she's baking and selling loads of yummy cookies and brownies. (Yes, it always smells great in my house.) She's hired a few more workers and our financial analyst has told us that the profit function is estimated by $P(x,y)=10x+6.4y-0.001x^2-0.025y^2$ where x is the number of cookies sold and y is the number of brownies sold. Find $\frac{\partial P}{\partial y}$ and give the approximate rate of change of profit with respect to the number of brownies sold if 100 cookies and 64 brownies are currently being sold. What does this mean?