Math1100 Final Review Problems

From all sections covered in class in chapters 9-14

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Overarching topics list:

Limits	Derivatives	Integrals
Different types of limits • plug in number and it's done case • 0/0 case • nonzero/0 case • infinity/infinity case Continuity • locate on a graph where a function is continuous vs. discontinuous • determine domain of a function	Find derivatives using short-cut rules • power rule • log/exponential rule • product rule • quotient rule • chain rule Higher-order derivatives Find equation of tangent line to curve at a given x-value (or point) Implicit differentiation Analyzing/sketching a graph using first and second derivative information • asymptotes • 1st derivative sign line • min/max points • 2nd derivative sign line • inflection points and concavity • sketch graph Story problems • optimization (minimize or maximize something like profit/cost/revenue) • related rates Functions of more than one variable • domain • evaluate at a point Partial derivatives	Indefinite integrals • power rule • algebra simplification first before doing integral • u-substitution Area • area between a curve and the x-axis • area between two curves Applications of definite integral • Total income from continuous income stream • Finding Cost, Revenue or Profit function given marginal cost, revenue or profit function • Present value and future value of income stream • Consumers and Producers surplus

1. Compute the following limits.

(a) $\lim_{x \to \infty} \frac{3x^2+1}{5x^2+x+5}$

(b) $\lim_{x \to \infty} \frac{3x+1}{5x^2+x+5}$

(c) $\lim_{x \to \infty} \frac{3x^3+1}{5x^2+x+5}$

(d) $\lim_{x \to 5} \frac{3x^2-6x-45}{2x^2-9x-5}$

(e) $\lim_{x \to 1} \frac{x^2+x-2}{x^2+3x-4}$

(f) $\lim_{x \to 2} \frac{x+9}{x^2-4}$

(g) $\lim_{x \to 2} \frac{x^2-4}{x+9}$

(h) $\lim_{x \to 1} \frac{x^2+x-2}{x^2+4\text{x}-5}$

(i) $\lim_{x \to \infty} \frac{ex^2-2\text{x}^3}{14\text{x}-3x^3}$

2. Find the derivative, $\frac{dy}{dx}$ of the following functions.

(a) $y=x^4-5x^{-3}+7-3^x+\ln x$

(b) $y=\frac{x^2-6x+9}{\ln x+5x}$

(c) $y=(x-7x^5)^9(e^{x^2+3x})$

(d) $\sqrt{x^2-9x}=e^y+\frac{1}{5}x^2$

(e) $x^3+8=\ln(xy)$

(f) $y=\sqrt[3]{5x+\ln(x^2+x^3)}$

(g) $y=3^{x^4}+e^{x^4}+x^{4e}$

(h) $y=\dfrac{(7x^4+e^{x^3}+2x)^7}{\sqrt[3]{2^x+x^5}}$

3. Find the equation of the tangent line for the given curve at the indicated point.

(a) $x^3+xy+4=0$ at $(2,-6)$

(b) $(x+2y)e^{xy}=xy^2$ at $(0,0)$

(c) $3x^2-4xy+2y^3=2x+16$ at (0, 2)

(d) $f(x)=x^2-3x$ at x = 2

(e) $f(x)=x^2-3x$ at x=1

4. Find $f^{(4)}(x)$ if $f(x)=\dfrac{1}{x^2}+7x^3-e^x+5x$.

5. Find the area between the two given curves.

(a) $y=6-x^2$ and $y=x$

(b) $y=4x+3$ and $y=x^2+3$

(c) $y=3x+2$ and $y=x^2+2$

(d) $y=8-x^2$ and $y=x^2$

6. Given the function $f(x,y)=\dfrac{7x-4y^2}{\sqrt{5x}}$

(a)State the domain of the function.

(b) Evaluate the function at (2,1).

7. The cost of producing x microwave ovens is $C(x)=0.01x^2+20x+300$ dollars, and the revenue function for the product is $R(x)=164x$.

(a) What is the profit function?

(b) How many microwave ovens should be sold to maximize profit?

(c) What is the maximum profit?

8. Compute the following integrals.

(a) $\int (2x^2 - x^4 - 5x^3 + 9)\,dx$

(b) $\int \left(\frac{5}{x^2} + e^x - \frac{2}{x}\right) dx$

(c) $\int 3x\,e^{x^2+5}\,dx$

(d) $\int \frac{x^3 + 4x - x^{-1}}{x}\,dx$

(e) $\int 100\,e^{-0.5x}\,dx$

(f) $\int (3x^2 - 8x + 2)^9 (3x - 4)\,dx$

(g) $\int \frac{2x^2}{x^3 - 1}\,dx$

(h) $\int \frac{-4}{2\mathrm{x} - 5}\,dx$

(i) $\int_1^3 (4x - 6x^2)\,dx$

(j) $\int_1^5 (3x^3 + 2\mathrm{x} - \frac{5}{x^2})\,dx$

(k) $\int_1^4 (6\mathrm{x}^2 + x - \frac{5}{x^2})\,dx$

(l) $\int_1^2 (4x^3 + 5\mathrm{x} - \frac{6}{x^3})\,dx$

(m) $\int_3^3 \ln x\,dx$

(n) $\int_0^3 x(8x^2 + 9)^{-1/2}\,dx$

9.For the function $y=x^3-2x^2+x+1$, answer the following questions.

(a) Find the horizontal and vertical asymptotes, if there are any.

(b) Fill in the first derivative sign line and find the min/max points.

(c) Fill in the second derivative sign line and find the inflection points.

(d) Sketch the graph of this function, given all the answers for questions (a) through (c).

10. For the function $y=x^4-2x^3+x^2$, answer the following questions.

(a) Find the horizontal and vertical asymptotes, if there are any.

(b) Fill in the first derivative sign line and find the min/max points.

(c) Fill in the second derivative sign line and find the inflection points.

(d) Sketch the graph of this function, given all the answers for questions (a) through (c).

11. For the function $f(x)=\frac{x^2-2x+5}{(x-3)^2}$ with $f'(x)=\frac{-4(x+1)}{(x-3)^3}$ and $f''(x)=\frac{14x+27}{(x-3)^4}$, answer the following questions.

(a) Find the horizontal and vertical asymptotes, if there are any.

(b) Fill in the first derivative sign line and find the min/max points.

(c) Fill in the second derivative sign line and find the inflection points.

(d) Sketch the graph of this function, given all the answers for questions (a) through (c).

12. Suppose the revenue of a company can be modeled by the function $R(x)=32x-0.05x^2$ where R(x) is the revenue in thousands of dollars from the sale of x thousand units of products.

(a) Find the marginal revenue function, $\overline{MR}(x)$.

(b) How many units should be sold to maximize revenue?

(c) What is the maximum revenue?

(d) If the production is limited to 250 units, how many units will maximize the total revenue?

(e) Write in words what $\overline{MR}(10)$ means.

13. A farmer has 200 feet of fencing and wishes to construct two pens for his animals by first building a fence around a rectangular region, and then subdividing that region into two smaller rectangles by placing a fence parallel to one of the sides. What dimensions of the region will maximize the total area?

14. For the function $g(x,y,z)=x^2 y e^z$ find $\frac{\partial g}{\partial x}+\frac{\partial g}{\partial y}+\frac{\partial g}{\partial z}$.

15. If the consumption is $8 billion when disposable income is 0, and if the marginal propensity to save is $\frac{dS}{dy}=0.5+e^{2.3y}$ (in billions of dollars), find the national consumption function.

16. Given $f(x,y)=\ln(x^2+2y)+x^4-2y^3+xy$ find the following partial derivatives.

(a) f_x

(b) f_y

(c) f_{xy}

(d) f_{xx}

(e) f_{yy}

17. For the function given by $f(x,y,z)=2xyz^2+x^3y^2z-y^4$ find the following partial derivatives.

(a) f_x

(b) f_{xz}

(c) f_{xyz}

(d) f_y when $x=1, y=0$ and $z=2$

18. Suppose that a product has marginal revenue given by $\overline{MR}=75$ and marginal cost given by $\overline{MC}=40+\frac{5}{2}x$. If the fixed cost is $105, how many units will give the maximum profit and what is the maximum profit?

19. Given the function $f(x,y,z)=\frac{2x^2+\ln z}{\sqrt{2y+6}}$ answer the following questions.

(a) Evaluate $f(1,5,1)$.

(b) Find the domain of $f(x,y,z)$.

20. A certain firm's marginal cost for a product is $\overline{MC}=5x+100$ and its marginal revenue is $\overline{MR}=180-2x$. The total profit of the production of 100 items is $15,000.

(a) Find the total profit function.

(b) Determine the level of production that yields the maximum profit.

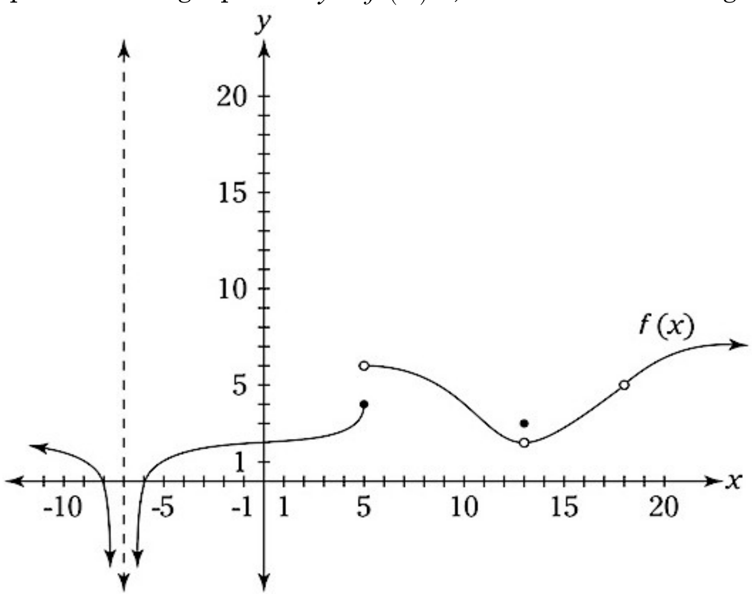
21. If $1000 is invested for x years at 8% compounded continuously, the future value of the investment is given by $S(x) = 1000e^{0.08x}$.

(a) Find the function that gives the rate of change of this investment.

(b) Compare the rate at which the future value is growing after 1 year and after 10 years.

22. The marginal cost for a product is $\overline{MC} = 12x + 20$ dollars per unit, and the cost of producing 50 items is $1,300. Find the total cost function.

23. If the graph below represents the graph of $y = f(x)$, answer the following questions.



(a) $\lim\limits_{x \to -6} f(x)$

(b) $\lim\limits_{x \to 5^+} f(x)$

(c) $\lim\limits_{x \to 5^-} f(x)$

(d) $\lim\limits_{x \to 5} f(x)$

(e) $\lim\limits_{x \to 13} f(x)$

(f) $\lim\limits_{x \to 18} f(x)$

(g) $f(-6)$

(h) $f(5)$

(i) $f(13)$

(j) $f(18)$

(k) For what x-values is $y = f(x)$ discontinuous?

24. Suppose a continuous income stream has an annual rate of flow $f(t) = 85e^{-0.01t}$, in thousands of dollars per year, and the current interest rate is 7% compounded continuously.

(a) Find the total income over the next 12 years.

(b) Find the present value over the next 12 years.

(c) Find the future value 12 years from now.

25. Suppose the supply function for a product is $p = 40 + 0.001x^2$ and the demand function is $p = 120 - 0.2x$, where x is the number of units and p is the price in dollars. If the market equilibrium price is $80, find the following.

(a) the consumer's surplus

(b) the producer's surplus

26. The cost of producing x cupcakes is given by $C(x) = 100 + 20x + 0.01x^2$ dollars. How many units should be produced to minimize average cost?

27. The demand function for a product under competition is $p = \sqrt{64 - 4x}$ and the supply function is $p = x - 1$, where x is the number of units and p is in dollars. Find the following.

(a) the market equilibrium point

(b) the consumer's surplus at market equilibrium

(c) the producer's surplus at market equilibrium