Math1100 Final Review Problems

From all sections covered in class in chapters 9-14
Fall, 2019

1. Compute the following limits.

(a) $\lim\limits_{x\to\infty}\frac{3x^2+1}{5x^2+x+5} = \lim\limits_{x\to\infty}\frac{3x^2}{5x^2} = \boxed{\frac{3}{5}}$

(b) $\lim\limits_{x\to\infty}\frac{3x+1}{5x^2+x+5} = \lim\limits_{x\to\infty}\frac{3x}{5x^2} = \lim\limits_{x\to\infty}\frac{3}{5x} = \boxed{0}$

(c) $\lim\limits_{x\to\infty}\frac{3x^3+1}{5x^2+x+5} = \lim\limits_{x\to\infty}\frac{3x^3}{5x^2} = \lim\limits_{x\to\infty}\frac{3x}{5} = \boxed{\infty \text{ or DNE}}$

(d) $\lim\limits_{x\to 5}\frac{3x^2-6x-45}{2x^2-9x-5} = \lim\limits_{x\to 5}\frac{\cancel{(x-5)}(3x+9)}{\cancel{(x-5)}(2x+1)} = \lim\limits_{x\to 5}\frac{3x+9}{2x+1} = \boxed{\frac{24}{11}}$

$\left(\frac{0}{0}\text{ case}\right)$

(e) $\lim\limits_{x\to 1}\frac{x^2+x-2}{x^2+3x-4} = \lim\limits_{x\to 1}\frac{\cancel{(x-1)}(x+2)}{\cancel{(x-1)}(x+4)} = \lim\limits_{x\to 1}\frac{x+2}{x+4} = \boxed{\frac{3}{5}}$

$\left(\frac{0}{0}\text{ case}\right)$

(f) $\lim\limits_{x\to 2}\frac{x+9}{x^2-4}$ $\boxed{\text{DNE}}$

$\left(\frac{11}{0}\text{ case}\right)$

(g) $\lim\limits_{x\to 2}\frac{x^2-4}{x+9} = \frac{0}{11} = \boxed{0}$

(h) $\lim\limits_{x\to 1}\frac{x^2+x-2}{x^2+4x-5} = \lim\limits_{x\to 1}\frac{\cancel{(x-1)}(x+2)}{\cancel{(x-1)}(x+5)} = \lim\limits_{x\to 1}\frac{x+2}{x+5} = \frac{3}{6}$

$\left(\frac{0}{0}\text{ case}\right)$

$= \boxed{\frac{1}{2}}$

(i) $\lim_{x\to\infty} \frac{e x^2 - 2x^3}{14x - 3x^3}$ $= \lim_{x\to\infty} \frac{-2x^3}{-3x^3} = \boxed{\frac{2}{3}}$

2. Find the derivative, $\frac{dy}{dx}$ of the following functions.

(a) $y = x^4 - 5x^{-3} + 7 - 3^x + \ln x$

$$\frac{dy}{dx} = \boxed{y' = 4x^3 + 15x^{-4} - 3^x(\ln 3) + \frac{1}{x}}$$

(b) $y = \frac{x^2 - 6x + 9}{\ln x + 5x}$

$$\frac{dy}{dx} = \boxed{y' = \frac{(\ln x + 5x)(2x-6) + (x^2-6x+9)(\frac{1}{x}+5)}{(\ln x + 5x)^2}}$$

(c) $y = (x - 7x^5)^9 (e^{x^2+3x})$

$$\boxed{y' = (x-7x^5)^9 (e^{x^2+3x}(2x+3)) + 9(x-7x^5)^8(1-35x^4)(e^{x^2+3x})}$$

need implicit differentiation

(d) $\sqrt{x^2 - 9x} = e^y + \frac{1}{5}x^2$

$$\frac{1}{2}(x^2-9x)^{-1/2}(2x-9) = e^y\left(\frac{dy}{dx}\right) + \frac{2}{5}x \iff \boxed{\frac{dy}{dx} = \frac{\frac{2x-9}{2\sqrt{x^2-9x}} - \frac{2}{5}x}{e^y}}$$

(e) $x^3 + 8 = \ln(xy)$

$$3x^2 = \frac{1}{xy}\left(y + x\frac{dy}{dx}\right) \iff \boxed{\frac{dy}{dx} = \frac{3x^3y - y}{x}}$$

(f) $y = \sqrt[3]{5x + \ln(x^2 + x^3)}$

$$\boxed{y' = \frac{1}{3}(5x + \ln(x^2+x^3))^{-2/3}\left(5 + \frac{2x+3x^2}{x^2+x^3}\right)}$$

(g) $y = 3^{x^4} + e^{x^4} + x^{4e}$

$$\boxed{y' = 3^{x^4}(\ln 3)(4x^3) + e^{x^4}(4x^3) + 4e x^{4e-1}}$$

(h) $y = \frac{(7x^4 + e^{x^3} + 2x)^7}{\sqrt[3]{2^x + x^5}}$

$$\boxed{y' = \frac{\sqrt[3]{2^x+x^5}\left(7(7x^4+e^{x^3}+2x)^6(28x^3+e^{x^3}(3x^2)+2)\right) - (7x^4+e^{x^3}+2x)^7 \Bigl(\frac{1}{3}(2^x+x^5)^{-2/3}(2^x\ln 2+5x^4)}{\left(\sqrt[3]{2^x+x^5}\right)^2}}$$

3. Find the equation of the tangent line for the given curve at the indicated point.

(a) $x^3+xy+4=0$ at $(2,-6)$

derivative: $3x^2+y+xy'=0$ ⟹ slope at $(2,-6)$

$y'=\frac{-3x^2-y}{x}$

$m=\frac{-3(2^2)-(-6)}{2}=-3$

$m=-3$, $(2,-6)$

line: $y+6=-3(x-2) \iff \boxed{y=-3x}$

(b) $(x+2y)e^{xy}=xy^2$ at $(0,0)$

derivative: $(1+2y')e^{xy}+(x+2y)e^{xy}(y+xy')=y^2+x(2y)y'$

plug in $(0,0)$ to derivative: $(1+2y')e^0+(0+0)e^0(0+0)=0+0$

$\iff 1+2y'=0 \iff y'=-1/2=$ slope

pt $(0,0)$, $m=-\frac{1}{2}$ ⟹ line: $y-0=-\frac{1}{2}(x-0) \iff \boxed{y=-\frac{1}{2}x}$

(c) $3x^2-4xy+2y^3=2x+16$ at $(0, 2)$

derivative: $6x-(4y+4xy')+6y^2y'=2$

plug in $(0,2)$: $6(0)-(4(2)+0(y'))+6(2^2)y'=2$

$\iff -8+24y'=2 \iff y'=\frac{5}{12}=$ slope

pt $(0,2)$, slope $m=\frac{5}{12}$ ⟹ line $\boxed{y=\frac{5}{12}x+2}$

(d) $f(x)=x^2-3x$ at x = 2

derivative: $f'(x)=2x-3$ slope: $f'(2)=2(2)-3=1=m$

pt: $(2,-2)$ $f(2)=2^2-3(2)=-2$

⟹ line is $y-(-2)=1(x-2) \iff \boxed{y=x-4}$

(e) $f(x)=x^2-3x$ at x=1

(same fn as (d)) $f'(x)=2x-3$

$m=\text{slope}=f'(1)=2(1)-3=-1$

pt $(1,-2)$ $f(1)=1^2-3(1)=-2$

⟹ line is $y+2=-1(x-1) \iff \boxed{y=-x-1}$

4. Find $f^{(4)}(x)$ if $f(x)=\frac{1}{x^2}+7x^3-e^x+5x$.

$$f(x)=x^{-2}+7x^3-e^x+5x$$

$$f'(x)=-2x^{-3}+21x^2-e^x+5$$

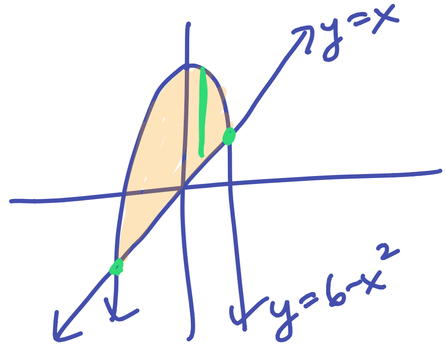
$$f''(x)=6x^{-4}+42x-e^x$$

$$f'''(x)=-24x^{-5}+42-e^x$$

$$\boxed{f^{(4)}(x)=120x^{-6}-e^x}$$

5. Find the area between the two given curves.

(a) $y=6-x^2$ and $y=x$



intersectn pts :

$$x=6-x^2$$

$$x^2+x-6=0$$

$$(x+3)(x-2)=0$$

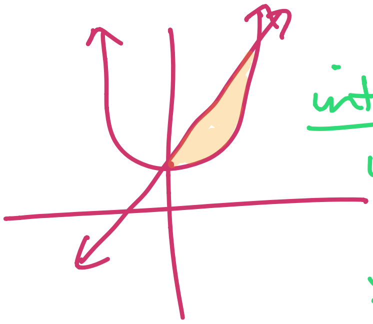
$$x=-3,2$$

$$A=\int_{-3}^{2}(6-x^2-x)\,dx=\left(6x-\frac{x^3}{3}-\frac{x^2}{2}\right)\Big|_{-3}^{2}$$

$$=\left(6(2)-\frac{8}{3}-2\right)-\left(-18+9-\frac{9}{2}\right)$$

$$=\boxed{\frac{125}{6}}$$

(b) $y=4x+3$ and $y=x^2+3$



intersection pts:

$$4x+3=x^2+3$$

$$x^2-4x=0$$

$$x(x-4)=0$$

$$x=0,4$$

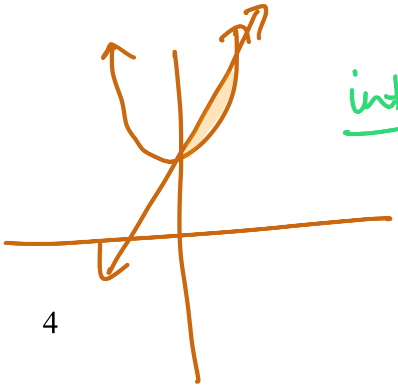
$$A=\int_{0}^{4}(4x+3-(x^2+3))\,dx$$

$$=\int_{0}^{4}(4x-x^2)\,dx$$

$$=\left(2x^2-\frac{x^3}{3}\right)\Big|_{0}^{4}$$

$$=2(16)-\frac{64}{3}-0=\boxed{\frac{32}{3}}$$

(c) $y=3x+2$ and $y=x^2+2$



intersection pts:

$$3x+2=x^2+2$$

$$3x=x^2$$

$$x^2-3x=0$$

$$x(x-3)=0$$

$$x=0,3$$

$$A=\int_{0}^{3}(3x+2-(x^2+2))\,dx$$

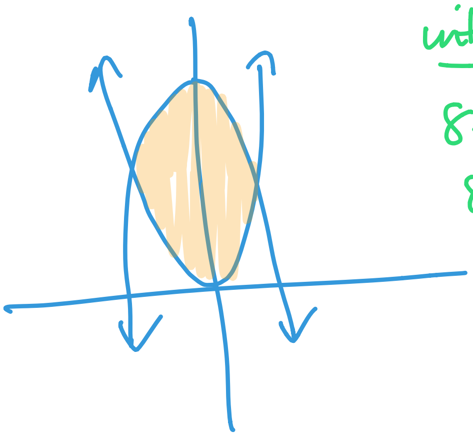
$$=\int_{0}^{3}(3x-x^2)\,dx$$

$$=\left(\frac{3x^2}{2}-\frac{x^3}{3}\right)\Big|_{0}^{3}$$

$$=\left(\frac{27}{2}-9\right)-0=\boxed{\frac{9}{2}}$$

5. Find the area between the two given curves.

(d) $y=8-x^2$ and $y=x^2$



intersection pts!

$8-x^2=x^2$

$8=2x^2$

$x^2=4$

$x=\pm 2$

$$A=\int_{-2}^{2}(8-x^2-x^2)\,dx$$

$$=\int_{-2}^{2}(8-2x^2)\,dx$$

$$=\left(8x-\frac{2x^3}{3}\right)\Big|_{-2}^{2}$$

$$=\left(16-\frac{16}{3}\right)-\left(-16+\frac{16}{3}\right)$$

$$=32-\frac{32}{3}=\boxed{\frac{64}{3}}$$

6. Given the function $f(x,y)=\dfrac{7x-4y^2}{\sqrt{5x}}$

(a)State the domain of the function.

$\boxed{x>0,\ y\in\mathbb{R}}$

(b) Evaluate the function at (2,1).

$$f(2,1)=\frac{7(2)-4(1^2)}{\sqrt{5(2)}}=\frac{10}{\sqrt{10}}\left(\frac{\sqrt{10}}{\sqrt{10}}\right)=\frac{10\sqrt{10}}{10}=\boxed{\sqrt{10}}$$

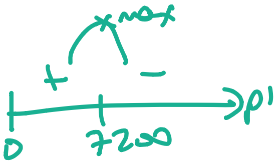
7. The cost of producing x microwave ovens is $C(x)=0.01x^2+20x+300$ dollars, and the revenue function for the product is $R(x)=164x$.

(a) What is the profit function?

$$P=R-C=164x-(0.01x^2+20x+300)=\boxed{-0.01x^2+144x-300=P(x)}$$

(b) How many microwave ovens should be sold to maximize profit?

$P'(x)=-0.02x+144=0$



$$x=\frac{144}{0.02}=\frac{14400}{2}=\boxed{7200}$$

(c) What is the maximum profit?

$$P(7200)=-0.01(7200^2)+144(7200)-300$$

$$=\boxed{\$518{,}100}$$

8. Compute the following integrals.

(a) $\int (2x^2 - x^4 - 5x^3 + 9)\,dx$

$$= \boxed{\frac{2x^3}{3} - \frac{x^5}{5} - \frac{5x^4}{4} + 9x + C}$$

(b) $\int \left(\frac{5}{x^2} + e^x - \frac{2}{x}\right)dx$ $= \int \left(5x^{-2} + e^x - \frac{2}{x}\right)dx$

$$= \frac{5x^{-1}}{-1} + e^x - 2\ln|x| + C = \boxed{\frac{-5}{x} + e^x - 2\ln|x| + C}$$

(c) $\int 3xe^{x^2+5}\,dx$

$u = x^2 + 5$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$= 3\left(\frac{1}{2}\right)\int e^u\,du$$

$$= \frac{3}{2}e^u + C$$

$$= \boxed{\frac{3}{2}e^{x^2+5} + C}$$

(d) $\int \frac{x^3 + 4x - x^{-1}}{x}\,dx$

$$= \int (x^2 + 4 - x^{-2})\,dx = \frac{x^3}{3} + 4x - \frac{x^{-1}}{-1} + C$$

$$= \boxed{\frac{1}{3}x^3 + 4x + \frac{1}{x} + C}$$

(e) $\int 100e^{-0.5x}\,dx$

$u = -0.5x$

$du = -0{,}5\,dx$

$-2\,du = dx$

$$= 100\int e^u(-2)\,du$$

$$= -200\int e^u\,du = -200e^u + C$$

$$= \boxed{-200e^{-0{,}5x} + C}$$

(f) $\int (3x^2 - 8x + 2)^9 (3x - 4)\,dx$

$u = 3x^2 - 8x + 2$

$du = (6x - 8)\,dx$

$\frac{1}{2}du = (3x - 4)\,dx$

$$= \frac{1}{2}\int u^9\,du = \frac{1}{2}\left(\frac{u^{10}}{10}\right) + C$$

$$= \boxed{\frac{1}{20}(3x^2 - 8x + 2)^{10} + C}$$

(g) $\int \frac{2x^2}{x^3-1}\,dx$ $= 2\left(\frac{1}{3}\right)\int \frac{1}{u}\,du = \frac{2}{3}\ln|u|+C$

$u = x^3-1$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2dx$

$$= \boxed{\frac{2}{3}\ln|x^3-1|+c}$$

(h) $\int \frac{-4}{2x-5}\,dx$ $= -2\int \frac{du}{u} = -2\int \frac{1}{u}\,du$

$u = 2x-5$

$du = 2dx$

$-2du = 4dx$

$$= -2\ln|u|+c$$

$$= \boxed{-2\ln|2x-5|+C}$$

(i) $\int_1^3 (4x-6x^2)\,dx$

$$= (2x^2-2x^3)\Big|_1^3 = (2(9)-2(27))-(2(1)-2(1))$$

$$= 18-54-0 = \boxed{-36}$$

(j) $\int_1^5 (3x^3+2x-\frac{5}{x^2})\,dx$ $= \int_1^5 (3x^3+2x-5x^{-2})\,dx$

$$= \left(\frac{3x^4}{4}+x^2-\frac{5x^{-1}}{-1}\right)\Big|_1^5 = \left(\frac{3}{4}x^4+x^2+\frac{5}{x}\right)\Big|_1^5$$

$$= \left(\frac{3}{4}(625)+25+1\right)-\left(\frac{3}{4}+1+5\right) = \boxed{488}$$

(k) $\int_1^4 (6x^2+x-\frac{5}{x^2})\,dx$ $= \int_1^4 (6x^2+x-5x^{-2})\,dx$

$$= \left(2x^3-\frac{x^2}{2}+\frac{5}{x}\right)\Big|_1^4 = \left(2(64)-\frac{16}{2}+\frac{5}{4}\right)-\left(2-\frac{1}{2}+5\right)$$

$$= \boxed{114.75}$$

(l) $\int_1^2 (4x^3+5x-\frac{6}{x^3})\,dx$ $= \int_1^2 (4x^3+5x-6x^{-3})\,dx$

$= (x^4 + \frac{5x^2}{2} - \frac{6x^{-2}}{-2})\Big|_1^2 = (x^4+\frac{5}{2}x^2+\frac{3}{x^2})\Big|_1^2$

$= (16 + \frac{5}{2}(4) + \frac{3}{4}) - (1 + \frac{5}{2} + 3) = \boxed{20.25}$

(m) $\int_3^3 \ln x\,dx$ $= \boxed{0}$ (you don't need to do this integral; it asks for area under curve with $\underline{\underline{0}}$ width, i.e. from $x=3$ to $x=3$ ⇒ area is 0)

(n) $\int_0^3 x(8x^2+9)^{-1/2}\,dx$

$u = 8x^2+9$

$du = 16x\,dx$

$\frac{1}{16}du = x\,dx$

$= \frac{1}{16}\int_{x=0}^{x=3} u^{-1/2}\,du$

$= \frac{1}{16}(u^{1/2}(2))\Big|_{x=0}^{x=3} = \frac{1}{8}(8x^2+9)^{1/2}\Big|_0^3 = \frac{1}{8}\sqrt{8x^2+9}\Big|_0^3$

$= \frac{1}{8}(9+3) = \boxed{\frac{3}{2}}$

9.For the function $y = x^3 - 2x^2 + x + 1$, answer the following questions.

(a) Find the horizontal and vertical asymptotes, if there are any.

$\boxed{\text{none}}$

$y(1/3) = \frac{1}{27} - \frac{2}{9} + \frac{1}{3} + 1$
$= 31/27$

$y(1) = 1-2+1+1$
$= 1$

(b) Fill in the first derivative sign line and find the min/max points.

$y' = 3x^2 - 4x + 1 = (3x-1)(x-1) = 0 \iff x = 1/3, 1$

+ − + y'

$\frac{1}{3}$ 1

/ \ /

⇒ min pt $(1, 1)$

max pt $(\frac{1}{3}, 31/27)$

(c) Fill in the second derivative sign line and find the inflection points.

$y'' = 6x - 4 = 0 \iff x = \frac{2}{3}$

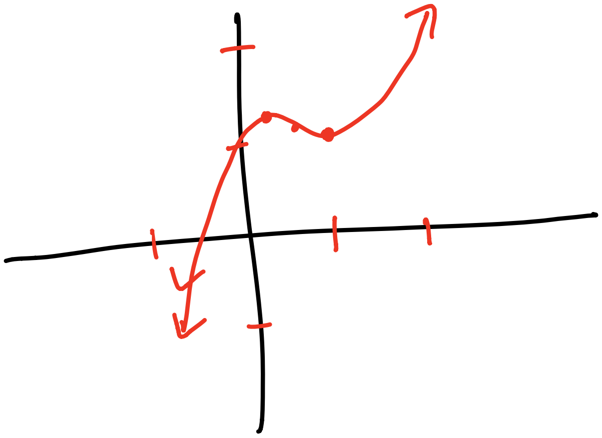
$y(\frac{2}{3}) = \frac{8}{27} - \frac{8}{9} + \frac{2}{3} + 1 = \frac{29}{27}$

− + y''

∩ 2/3 ∪

inflection pt: $(\frac{2}{3}, \frac{29}{27})$

(d) Sketch the graph of this function, given all the answers for questions (a) through (c).



10. For the function $y=x^4-2x^3+x^2$, answer the following questions.

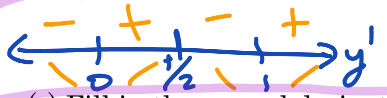
(a) Find the horizontal and vertical asymptotes, if there are any.

none

(b) Fill in the first derivative sign line and find the min/max points.

$y' = 4x^3-6x^2+2x = 2x(2x^2-3x+1) = 2x(2x-1)(x-1) = 0$

$x = 0, 1/2, 1$



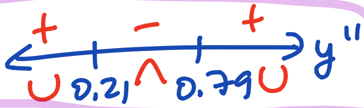
min pts: (0,0)(1,0)

max pt: $(\frac{1}{2}, \frac{1}{16})$

(c) Fill in the second derivative sign line and find the inflection points.

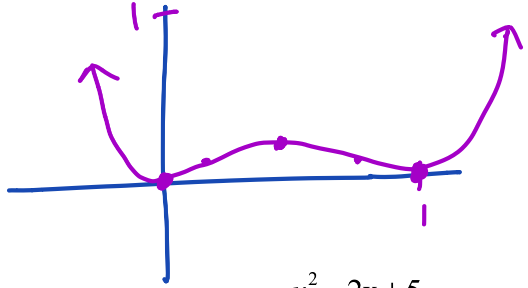
$y'' = 12x^2-12x+2 = 2(6x^2-6x+1) = 0 \iff x = \frac{6\pm\sqrt{36-4(6)}}{2(6)}$

$x \approx 0.79, 0.21$ $x = \frac{1}{2} \pm \frac{\sqrt{3}}{6}$



inflectn pts: $\sim(0.79, 0.03)$ & $(0.21, 0.03)$

(d) Sketch the graph of this function, given all the answers for questions (a) through (c).



11. For the function $f(x)=\frac{x^2-2x+5}{(x-3)^2}$ with $f'(x)=\frac{-4(x+1)}{(x-3)^3}$ and $f''(x)=\frac{14x+27}{(x-3)^4}$,

answer the following questions.

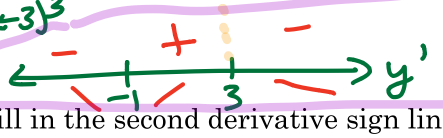
(a) Find the horizontal and vertical asymptotes, if there are any.

VA: x=3 HA: y=1

$\lim_{x\to\infty}\frac{x^2-2x+5}{(x-3)^2} = \lim_{x\to\infty}\frac{x^2}{x^2} = 1$

(b) Fill in the first derivative sign line and find the min/max points.

$\frac{-4(x+1)}{(x-3)^3} = 0$ when $x=-1$ also $x=3$ makes derivative undefined



min pt: $(-1, \frac{1}{2})$

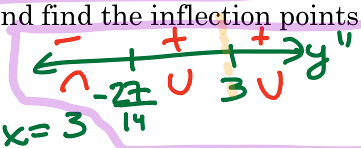
no maxpt.

$f(-1) = \frac{1+2+5}{16} = \frac{1}{2}$

(c) Fill in the second derivative sign line and find the inflection points.

$\frac{14x+27}{(x-3)^4} = 0$ when $x = \frac{-27}{14}$

and $f''(x)$ undefined @ $x=3$

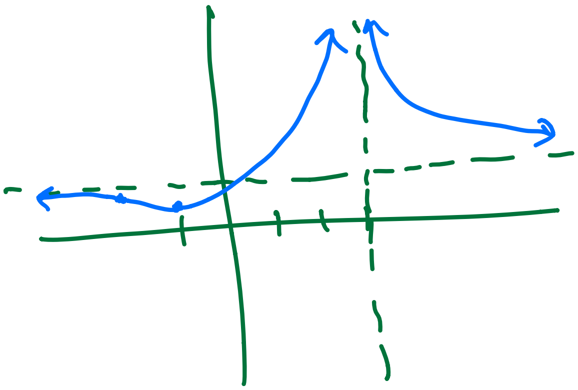


inflectn pt:

$\sim(\frac{-27}{14}, 0.52)$

$\approx(-1.93, 0.52)$

(d) Sketch the graph of this function, given all the answers for questions (a) through (c).



12. Suppose the revenue of a company can be modeled by the function $R(x)=32x-0.05x^2$ where R(x) is the revenue in thousands of dollars from the sale of x thousand units of products.

this is concave down parabola

⇒ vertex is max pt

(a) Find the marginal revenue function, $\overline{MR}(x)$.

$$\boxed{\overline{MR}(x) = 32 - 0.1x}$$

(b) How many units should be sold to maximize revenue?

$$32 - 0.1x = 0$$
$$0.1x = 32 \iff x = \boxed{320 \text{ thousand units}}$$

(c) What is the maximum revenue?

$$R(320) = 32(320) - 0.05(320^2) = 5120 \text{ thousand dollars}$$

i.e. $\boxed{\$\,5120{,}000}$

(d) If the production is limited to 250 thousand units, how many units will maximize the total revenue?

$$\boxed{250{,}000}$$

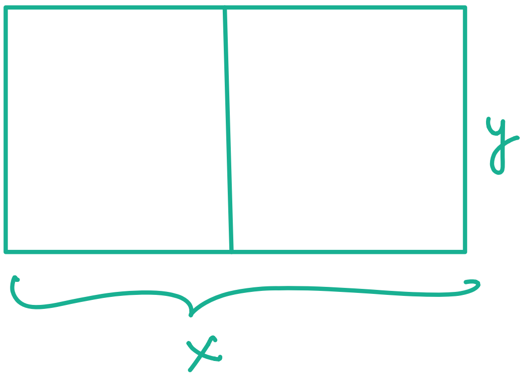
(e) Write in words what $\overline{MR}(10)$ means.

when x=10, the next 1000 units will cost $\overline{MR}(10)$ thousand dollars

→ since x is measured in 1000 units

13. A farmer has 200 feet of fencing and wishes to construct two pens for his animals by first building a fence around a rectangular region, and then subdividing that region into two smaller rectangles by placing a fence parallel to one of the sides. What dimensions of the region will maximize the total area?

x, y = ? to maximize Area



$$A = xy$$

200 ft of fencing ⇒

$$3y + 2x = 200$$
$$3y = 200 - 2x$$
$$y = \frac{200-2x}{3}$$

$$\Rightarrow A = x\left(\frac{200}{3} - \frac{2}{3}x\right) = \frac{200}{3}x - \frac{2}{3}x^2$$ → this is concave down parabola ⇒ vertex is max pt.

$$A'(x) = \frac{200}{3} - \frac{4}{3}x = 0 \iff x = 50$$

⇒ max area when $\boxed{x = 50\text{ft}}$

$$\Rightarrow y = \frac{200-2(50)}{3} = \boxed{\frac{100}{3}\text{ ft}}$$

14. For the function $g(x,y,z)=x^2ye^z$ find $\frac{\partial g}{\partial x}+\frac{\partial g}{\partial y}+\frac{\partial g}{\partial z}$ $= \boxed{2xye^z + x^2e^z + x^2ye^z}$

$\frac{\partial g}{\partial x} = 2xye^z, \quad \frac{\partial g}{\partial y} = x^2e^z, \quad \frac{\partial g}{\partial z} = x^2ye^z$

15. If the consumption is \$8 billion when disposable income is 0, and if the marginal propensity to save is $\frac{dS}{dy}=0.5+e^{2.3y}$ (in billions of dollars), find the national consumption function.

$\frac{dS}{dy} = 1 - \frac{dC}{dy} \Leftrightarrow \frac{dC}{dy} = 1 - \frac{dS}{dy} = 1-(0.5+e^{2{,}3y}) = 0.5 - e^{2{,}3y}$

$C = \int (0.5 - e^{2{,}3y})\,dy = 0{,}5y - \frac{e^{2{,}3y}}{2{,}3} + K$

but $C=8$ when $y=0 \Rightarrow 8 = 0 - \frac{1}{2{,}3} + K \Rightarrow K \simeq 8{,}43$

$\Rightarrow \boxed{C(y) = 0.5y - \frac{1}{2{,}3}e^{2{,}3y} + 8.43}$

16. Given $f(x,y)=\ln(x^2+2y)+x^4-2y^3+xy$ find the following partial derivatives.

(a) $f_x = \frac{2x}{x^2+2y} + 4x^3 + y = 2x(x^2+2y)^{-1} + 4x^3 + y$

(b) $f_y = \frac{2}{x^2+2y} - 6y^2 + x = 2(x^2+2y)^{-1} - 6y^2 + x$

(c) $f_{xy} = -2x(x^2+2y)^{-2}(2) + 1 = -4x(x^2+2y)^{-2} + 1$

(d) $f_{xx} = 2(x^2+2y)^{-1} + 2x(-1)(x^2+2y)^{-2}(2x) + 12x^2$

(e) $f_{yy} = -2(x^2+2y)^{-2}(2) - 12y$

17. For the function given by $f(x,y,z)=2xyz^2+x^3y^2z-y^4$ find the following partial derivatives.

(a) $f_x = \boxed{2yz^2 + 3x^2y^2z}$

(b) $f_{xz} = \boxed{4yz + 3x^2y^2}$

(c) $f_{xyz} = f_{xzy} = \boxed{4z + 6x^2y}$

(d) f_y when $x=1, y=0$ and $z=2$

$f_y = 2xz^2 + 2x^3yz - 4y^3$

$f_y(1,0,2) = 2(1)(2^2) + 2(1)(0)(2) - 4(0) = \boxed{8}$

18. Suppose that a product has marginal revenue given by $\overline{MR}=75$ and marginal cost given by $\overline{MC}=40+\frac{5}{2}x$. If the fixed cost is \$105, how many units will give the maximum profit and what is the maximum profit?

$\overline{MC}(x)=40+2.5x \Rightarrow C(x)=40x+1.25x^2+D$ but $C(0)=105$

$\Rightarrow D=105 \Rightarrow C(x)=40x+1.25x^2+105$

$\overline{MR}(x)=75 \Rightarrow R(x)=\int 75\,dx=75x+K$ but $R(0)=0$

$\Rightarrow K=0 \Rightarrow R(x)=75x$

$\Rightarrow P(x)=75x-(40x+1.25x^2+105)=-1.25x^2+35x-105$

& max pt

concave down parabola

$\Rightarrow P'(x)=-2.5x+35=0$

$x=14$

$\Rightarrow$ max profit at $\boxed{x=14}$

$P(14)=\boxed{\$140}$ max profit

19. Given the function $f(x,y,z)=\frac{2x^2+\ln z}{\sqrt{2y+6}}$ answer the following questions.

(a) Evaluate $f(1,5,1)$.

$$f(1,5,1)=\frac{2(1)+\ln 1}{\sqrt{10+6}}=\frac{2+0}{4}=\boxed{\frac{1}{2}}$$

(b) Find the domain of $f(x,y,z)$.

$z>0,\quad 2y+6>0 \Leftrightarrow y>-3$

$\boxed{x\in\mathbb{R},\ z>0,\ y>-3}$

20. A certain firm's marginal cost for a product is $\overline{MC}=5x+100$ and its marginal revenue is $\overline{MR}=180-2x$. The total profit of the production of 100 items is \$15,000.

(a) Find the total profit function.

$\overline{MP}=\overline{MR}-\overline{MC}=180-2x-5x-100=80-7x$

$\Rightarrow P(x)=\int(80-7x)\,dx=80x-3.5x^2+D$

$P(100)=15000 \Rightarrow 15000=80(100)-3.5(100^2)+D$

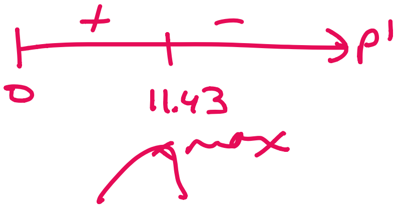
$\Leftrightarrow D=-12000$

$\Rightarrow \boxed{P(x)=80x-3.5x^2-12000}$

(b) Determine the level of production that yields the maximum profit.

$P'(x)=80-7x=0$

$x=\frac{80}{7}\simeq 11.43$



$\boxed{x=11.43}$ produces max profit

21. If \$1000 is invested for x years at 8% compounded continuously, the future value of the investment is given by $S(x)=1000e^{0.08x}$.

(a) Find the function that gives the rate of change of this investment.

$$\frac{dS}{dx}=S'=1000e^{0.08x}(0.08)=\boxed{80e^{0.08x}}$$

(b) Compare the rate at which the future value is growing after 1 year and after 10 years.

$$S'(1)=80e^{0.08}\approx\boxed{86.66}$$

$$S'(10)=80e^{0.8}\approx\boxed{178.04}$$

} big difference in rate of growth

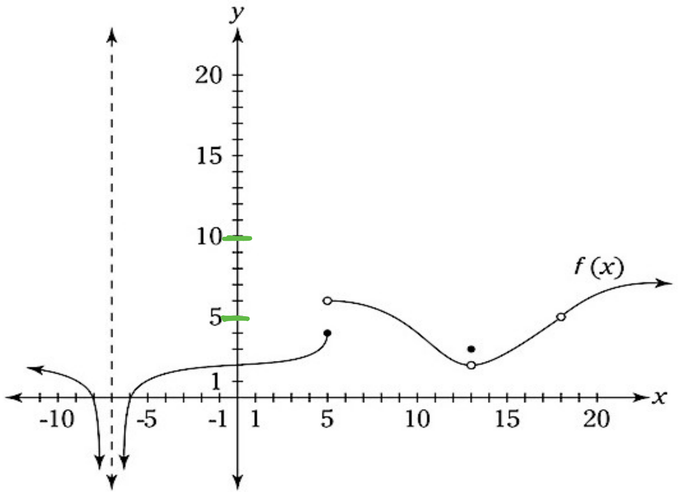
22. The marginal cost for a product is $\overline{MC}=12x+20$ dollars per unit, and the cost of producing 50 items is \$1,300. Find the total cost function.

$$C(x)=\int(12x+20)\,dx=6x^2+20x+D$$

$$C(50)=1300 \Rightarrow 6(50^2)+20(50)+D=1300 \Rightarrow D=-14700$$

$$\Rightarrow \boxed{C(x)=6x^2+20x-14700}$$

23. If the graph below represents the graph of $y=f(x)$, answer the following questions.



(a) $\lim_{x\to-6} f(x) = -\infty$ or DNE

(b) $\lim_{x\to5^+} f(x) = 6$

(c) $\lim_{x\to5^-} f(x) = 4$

(d) $\lim_{x\to5} f(x)$ DNE

(e) $\lim_{x\to13} f(x) = 2$

(f) $\lim_{x\to18} f(x) = 6$

(g) $f(-6)$ DNE

(h) $f(5) = 4$

(i) $f(13) = 3$

(j) $f(18)$ DNE

(k) For what x-values is $y=f(x)$ discontinuous?

@ $x=-6, 5, 13$

VA jump hole

24. Suppose a continuous income stream has an annual rate of flow $f(t)=85e^{-0.01t}$, in thousands of dollars per year, and the current interest rate is 7% compounded continuously.

(a) Find the total income over the next 12 years.

$$I = \int_0^{12} 85e^{-0.01t}\,dt = \frac{85e^{-0.01t}}{-0.01}\Big|_0^{12} = -8500e^{-0.01t}\Big|_0^{12}$$

$$= -8500(e^{-0.12} - e^{0}) = \boxed{\$961.18}$$

(b) Find the present value over the next 12 years.

$$PV = \int_0^{12} 85e^{-0.01t}e^{-0.07t}\,dt = 85\int_0^{12} e^{-0.08t}\,dt$$

$$= \frac{85}{-0.08}e^{-0.08t}\Big|_0^{12} = \frac{85}{-0.08}\left(e^{-0.08(12)} - e^{0}\right) \simeq \boxed{\$655.68} \downarrow$$

(c) Find the future value 12 years from now.

$$FV = e^{0.07(12)}\int_0^{12} 85e^{-0.01t}e^{-0.07t}\,dt = e^{0.07(12)}\left[655.68\right]$$

$$\simeq \boxed{\$1518.79}$$

25. Suppose the supply function for a product is $g(x)=$ $p=40+0.001x^2$ and the demand function is $f(x)=$ $p=120-0.2x$, where x is the number of units and p is the price in dollars. If the market equilibrium price is $80, find the following.

(a) the consumer's surplus

$$CS = \int_0^{x_1} f(x)\,dx - p_1x_1$$

$$= \int_0^{200}(120-0.2x)\,dx - 80(200)$$

$$= (120x - 0.1x^2)\Big|_0^{200} - 16000$$

$$= \boxed{\$4000}$$

$p_1 = 80$

$80 = 40 + 0.001x_1^2$

$40 = 0.001x_1^2$

$40{,}000 = x_1^2$

$200 = x_1$

(b) the producer's surplus

$$PS = p_1x_1 - \int_0^{x_1} g(x)\,dx = 80(200) - \int_0^{200}(40 + 0.001x^2)\,dx$$

$$= 16000 - \left(40x + \frac{0.001}{3}x^3\right)\Big|_0^{200}$$

$$= 16000 - \left(40(200) + \frac{0.001}{3}(200^3) - 0\right)$$

$$\simeq \boxed{\$5333.33}$$

26. The cost of producing x cupcakes is given by $C(x) = 100 + 20x + 0.01x^2$ dollars. How many units should be produced to minimize average cost?

$\bar{C} = \text{avg cost} = \frac{C(x)}{x} = \frac{100}{x} + 20 + 0.01x = 100x^{-1} + 20 + 0.01x$

$\bar{C}'(x) = \frac{-100}{x^2} + 0.01 = 0$

$\frac{1}{100} = \frac{100}{x^2} \iff x^2 = 100^2$

$\iff \boxed{x = 100}$ (since x can't be negative)

27. The demand function for a product under competition is $p = \overset{f(x)}{\sqrt{64 - 4x}}$ and the supply function is $p = x - 1$, where x is the number of units and p is in dollars. Find the following.

g(x) (a) the market equilibrium point

$\sqrt{64-4x} = x-1$

$64-4x = (x-1)^2$

$64-4x = x^2-2x+1$

$x^2+2x-63=0 \rightarrow (x+9)(x-7)=0$

$x = \cancel{-9}, 7$

equilibrium pt at $x=7$

$p = x-1 = 7-1 = 6$

$\boxed{(7, \$6)}$ equil. pt

(b) the consumer's surplus at market equilibrium

$CS = \int_0^{x_1} f(x)\,dx - px_1 = \int_0^7 \sqrt{64-4x}\,dx - 6(7)$

$u = 64-4x$
$du = -4\,dx$
$-\frac{1}{4}du = dx$

$= \int_{x=0}^{x=7} u^{1/2}\left(\frac{-1}{4}\right)du - 42 = \frac{-1}{4}u^{3/2}\left(\frac{2}{3}\right)\Big|_{x=0}^{x=7} - 42$

$= \frac{-1}{6}(64-4x)^{3/2}\Big|_0^7 - 42$

$= \frac{-1}{6}(216-512) - 42$

$= \boxed{\$7.33}$

(c) the producer's surplus at market equilibrium

$PS = p_1x_1 - \int_0^{x_1} g(x)\,dx$

$= 6(7) - \int_0^7 (x-1)\,dx = 42 - \left(\frac{x^2}{2} - x\right)\Big|_0^7$

$= 42 - \left(\frac{49}{2} - 7\right) = \boxed{\$24.50}$