ANSWER KEY

Math1100 Midterm 2 Review Problems
Sections 10.3-10.5, 11.1-11.4, 12.1-12.4
Fall, 2019
Instructor: Kelly MacArthur

1. Find the derivative, $\frac{dy}{dx}$ of the following functions.

(a) $y=5^x+\ln(x^2+3x)+3x^4$

(a) $y' = 5^x(\ln 5) + \frac{2x+3}{x^2+3x} + 12x^3$

(b) $y=\frac{x^4+\frac{2}{x}+\ln x}{3x+5}$

(b) $y' = \frac{(3x+5)\left(4x^3 + \frac{-2}{x^2} + \frac{1}{x}\right) - \left(x^4+\frac{2}{x}+\ln x\right)(3)}{(3x+5)^2}$

(c) $y=(e^{x^2+4x})(5x^{-2}+4x)^3$

(c) $y' = e^{x^2+4x}(2x+4)(5x^{-2}+4x)^3 + e^{x^2+4x}(3)(5x^{-2}+4x)^2(-10x^{-3}+4)$

(need implicit diff.)

(d) $\sqrt{x^3+x^4}=e^x+\frac{x^2}{y^3}$

(d) $y' = \frac{\frac{3x^2+4x^3}{2\sqrt{x^3+x^4}} - e^x - \frac{2x}{y^3}}{\frac{-3x^2}{y^4}}$

(e) $xe^{xy}=\ln(xy)$

(e) $y' = \frac{\frac{1}{x} - e^{xy} - xye^{xy}}{x^2e^{xy} - \frac{1}{y}}$

(f) $y=\frac{e^x-1}{e^x+1}$

(g) $y=2^{x^2}+e^{2x}$

1(g) $y' = 2^{x^2}(\ln 2)(2x) + e^{2x}(2)$

(f) $y' = \frac{(e^x+1)(e^x) - (e^x-1)e^x}{(e^x+1)^2} = \frac{2e^x}{(e^x+1)^2}$

2. Find $f'''(x)$ if $f'(x)=x\ln x$.

$f'''(x) = \frac{1}{x}$

3. Find the equation of the tangent line for the given curve at the indicated point.

(a) $xe^y=2y+3$ at (3, 0)

(a) $y = -x+3$

(b) $y=2^{3x+1}+(3x+1)^2$ when x = 0

(b) $y = 6(1+\ln 2)x + 3$

(c) $x^4+xy^4=2y^2+x-1$ at (-1, 1)

(c) $y = -\frac{1}{2}x + \frac{1}{2}$

(d) $x^2-4x+2y^2-4=0$ at (2, 2)

(d) $y = 2$

extraneous info.

4. Suppose a spherical snowball of volume 340 cubic feet is melting at a rate of 10 cubic feet per hour. As it melts, it remains spherical. At what rate is the radius changing ~~after 2.5 hours~~? when radius is 1 ft.

$dr/dt = \frac{-5}{2\pi}$ ft/hr

related rates

5. When the price of a certain commodity is p dollars per unit, customers demand x hundred units, where the relationship between demand and price is given by $x^2+4px+p^2=52$. How fast is demand, x, changing with respect to time when the price is \$2 per unit and is decreasing at a rate of \$0.30 per month?

$\frac{dx}{dt} = \frac{-3}{8}$ units/month

6. Compute the following integrals.

(a) $\int (x^2+4x^3-5x+1)\,dx$ $= \frac{x^3}{3} + x^4 - \frac{5x^2}{2} + x + C$

(b) $\int (\frac{3}{x}+e^x-\frac{2}{x^2})\,dx$ $= 3\ln|x| + e^x + 2x^{-1} + C$

(c) $\int \frac{2}{x\ln(5x)}\,dx$ $= 2\ln\left|\ln(5x)\right| + C$

let $u = \ln(5x)$

(d) $\int 3x(x^2+4)^{29}\,dx$ $= \frac{1}{20}(x^2+4)^{30} + C$

let $u = x^2+4$

(e) $\int \frac{x^3}{\sqrt{x^4+2}}\,dx$ $= \frac{1}{2}\sqrt{x^4+2} + C$

let $u = x^4+2$

(f) $\int \frac{x^2+4}{x}\,dx$ $= \int (x + \frac{4}{x})\,dx = \frac{x^2}{2} + 4\ln|x| + C$

(g) $\int (3x^2+6x+1)^7(x+1)\,dx$ $= \frac{1}{48}(3x^2+6x+1)^8 + C$

let $u = 3x^2+6x+1$

(h) $\int \frac{2x^3}{x^4+2}\,dx$ $= \frac{1}{2}\ln|x^4+2| + C$

let $u = x^4+2$

(i) $\int 3x^5(x^6-2)^{-3}\,dx$ $= \frac{-1}{4(x^6-2)^2} + C$

let $u = x^6-2$

7. A certain firm's marginal cost for a product is $\overline{MC}=6x+60$ and its marginal revenue is $\overline{MR}=180-2x$. The total profit of the production of 10 items is \$1000.

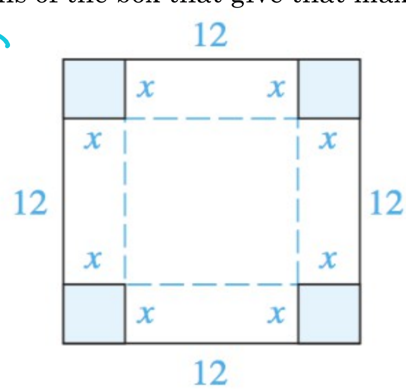
(a) Find the total profit function.

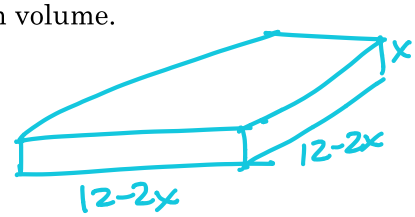
$P(x) = -4x^2 + 120x + 200$

(b) Determine the level of production that yields the maximum profit.

at $x = 15$ units

8. A square piece of cardboard 12 inches by 12 inches is to be formed into a box by cutting the squares of length x inches from each corner and folding up the sides. Find the maximum volume of the box and find the dimensions of the box that give that maximum volume.

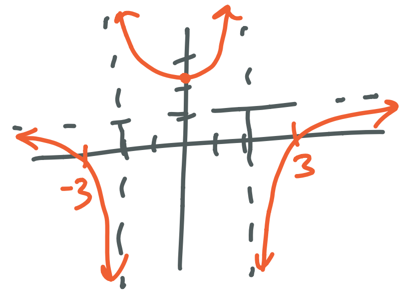




max Volume when $x = 2$ inches

$\Rightarrow$ box dimensions $8'' \times 8'' \times 2''$

and max volume $= 128\ in^3$



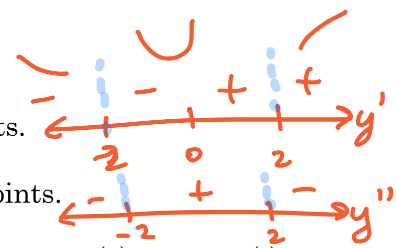
9. Consider the function $y=\frac{x^2-9}{x^2-4}$ with derivatives $y'=\frac{10x}{(x^2-4)^2}$ and $y''=\frac{-10(3x^2+4)}{(x^2-4)^3}$. Use these to answer the following questions.

(a) Find the horizontal and vertical asymptotes.

HA: $y=1$ VA: $x=2, x=-2$

(b) Fill in the first derivative sign line and find the min/max points.

no max / min pt at $(0, 9/4)$



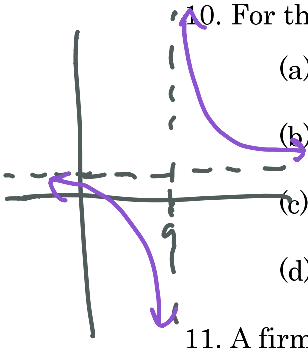
(c) Fill in the second derivative sign line and find the inflection points.

no inflectn pts

(d) Sketch the graph of this function, given all the answers for questions (a) through (c).

$1+\frac{3}{x-9}$

10. For the function $y=\frac{x-6}{x-9}$, answer the following questions. $y'=\frac{-3}{(x-9)^2}$, $y''=\frac{6}{(x-9)^3}$

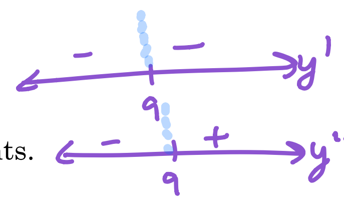


(a) Find the horizontal and vertical asymptotes.

HA: $y=1$ VA: $x=9$

(b) Fill in the first derivative sign line and find the min/max points.

no min/max pts



(c) Fill in the second derivative sign line and find the inflection points.

no inflectn pts

(d) Sketch the graph of this function, given all the answers for questions (a) through (c).

11. A firm can only produce 1000 units per month. Suppose the monthly total cost and monthly revenue functions, in dollars, are given by

$$C(x)=300+200x$$

$$R(x)=250x-\frac{1}{100}x^2,$$

where x is the number of units produced and sold. How many units, x, should the firm produce and sell for maximum profit? What is the maximum profit? $x=1000$ units

$P=\$39,700$

12. Suppose the revenue of a company can be modeled by the function $R(x)=50xe^{(1-0.02x)}$ where R(x) is the revenue in thousands of dollars from the sale of x thousand units of products. How many units should be sold to maximize revenue? What is the maximum revenue?

$x=50,000$ units $R=\$2500,000$

13. The cost of producing x microwave ovens is $C(x)=0.01x^2+20x+300$ dollars, and the revenue function for the product is $R(x)=164x$. How many microwave ovens should be sold to maximize profit? What is the maximum profit?

$x=7200$ ovens, max $P=\$518,100$

14. A young woman needs to rescue her friend from a cruel witch who has the friend locked in a room. The woman has placed a 20-foot ladder against the witch's house and climbed to the top of the ladder to knock on the window of her friend. The witch starts pulling the bottom of the ladder away from the house at a rate of 3 feet per second. How fast is the top of the ladder (and the young woman) falling when the bottom of the ladder is 12 feet from the bottom of the wall?

-2.25 ft/sec

15. If \$1000 is invested for n years at 12% compounded continuously, the future value of the investment is given by $S=1000e^{0.12n}$.

$\frac{ds}{dn} = 120e^{0.12n}$

(a) Find the function that gives the rate of change of this investment.

(b) Compare the rate at which the future value is growing after 1 year and after 10 years.

at $n=1$, $\frac{ds}{dn} = 120e^{0.12} \approx \$135.30/yr$ at $n=10$, $\frac{ds}{dn} = 120e^{1.2} \approx \$398.41/yr$

16. Disposable income is the amount available for spending and saving after taxes have been paid and is one gauge for the state of the economy. Using U.S. Energy Administration data for selected years from 2010 and projected to 2040, the total U.S. disposable income, in billions of dollars, can be modeled by $D(t) = 10{,}020e^{0.02292t}$ where t is the number of years past 2010.

(a) Find the function that models the rate of change of disposable income. $\frac{dD}{dt} = 229.6584e^{0.02292t}$

(b) Find and interpret D(20) and D'(20).

$D(20) \approx \$15847.05$ billion = disposable income in 2030

17. Suppose the demand function for a product is given by $(p+1)q^2 = 10{,}000$ where p is the price and q is the quantity demanded. Find the rate of change of quantity with respect to price when p = \$99.

$\frac{dq}{dp} = -0.05$

18. The marginal cost for a product is $\overline{MC} = 6x+4$ dollars per unit, and the cost of producing 100 items is \$31,400. Find the total cost function.

$C(x) = 3x^2 + 4x + 1000$

16(b) $D'(20) = \$363.21$ billion/yr = rate that disposable income is changing in 2030