Math2270 Project #2 Fall, 2019 Kelly MacArthur

Instructions:
Based on the survey results, students have been grouped according to your stated preferences. All students who filled out the survey were assigned either their first or second choice for assigned tasks within each group. Here are the final assignments.

Group #	Poster	Writing	Programming	Extra Person
1	Josh C.	Butros	Michael	
2	Hannah	Vig	Jon E.	
3	Alex N.	Andrew Moore	Ryan	
4	Brigham	Chris S.	Alex C.	
5	Anthony	Ardyn	Seamus	
6	Jacob K.	Rohun	Mitch	
7	Peter	Alex G.	Yerry	
8	Katherine	Carson	Nathan Gor...	
9	Alyssa	Sam	Nate	Krishna
10	Jacob P.	Chris B.	Brian	Avery
11	Nathan Gol....	Mateo	Jonathan W.	
12	Chucky	Ike	Josh M.	
14	Lauren	Jonathan O.	Kyle	
15	Marc	Noah	Anna	

General--

- **Due date**: You need to upload your completed project (according to the detailed instructions below) to the corresponding assignment **in Canvas by 11:59 pm on Sunday, December 1st**.

- The **poster presentation will be Wednesday, December 4th in class** where you will be required to bring and present your poster as a group.

- Submit your own group work, in your own words (i.e. not copied from somewhere online nor copied from any other group).

These projects are taken or adapted from https://sites.math.washington.edu/~marshall/math_136/projects/index.html.

Specific--

1. Choose one of the following projects to do (on the next several pages). (*Note: If there is some other application that your entire group really wants to do that's not on this list, you need to (a) be in consensus and (b) get my approval for a different idea by Monday, October 21 in order to pursue that other idea that you've created and submitted to me.*)

2. Every person in the group is responsible for getting the math done.
Additionally

(a) the assigned programmer is primarily responsible for writing up any code that needs to be done for this project. You need to do the programming in Matlab in order to get some exposure and experience using that tool, as it's incredibly powerful for linear algebra applications. The code should be included as an appendix to the written project. Also the code should be neat and organized and easy to read through, with some comments that help the reader follow what's happening.

(b) the assigned writer is primarily responsible for writing up the paper that describes the problems and the results. Please include the problem statements with the solutions. This report should be typed (you can use LaTeX or OpenOffice or Word, as they all have math type embedded there). There is no specific page requirement, just write as much as is necessary to finish the project. Cite any sources. Use 12-point font, Times New Roman and 1.5 line spacing, so it's readable for me.

(c) the assigned poster creator is primarily responsible for creating the poster to showcase the problems and solutions/results during class on Monday, December 2nd. The poster should visually represent what's in the code and the paper. Also, a pdf version of the poster needs to be included with the uploaded report as an appendix to the report.

3. (a) By Sunday, December 1st, you will upload ONE pdf file that contains the typed report with the following appendices: (1) the Matlab code and (2) any written math that had to be first worked out by hand (this third item may be scratch work and that's fine...I just want to see what went into completing the math).
(b) By Tuesday, December 3rd, you will upload ONE pdf file that contains a picture/pdf image of the poster you'll be presenting on that Wednesday in class.

4. Please be sure that each person in the group is explaining their work to the other group members and getting their input and feedback along the way. Also, since these tasks have to be done somewhat in a sequential order (math --> programming --> writing the report --> making the poster), please do NOT procrastinate and create an adverse domino effect for the people whose primary tasks come later in the process. Be respectful of all group members and the work they need to get done. If something is going awry, talk about it early so it can get addressed. If you're having any issues with that, please contact me and I can help navigate the challenges with you.

To this end, here is a good set of dates to use as goalposts for completing the project.

Date	Goal(s)
10/18/19, Friday	Decide on which project you will do. Be sure you know where/which computers you have access to Matlab.
11/15/19, Friday	Finish the math and coding for the problems.
11/22/19, Friday	Finish writing the report.
11/29/19, Friday	Finish the poster.
12/1/19, Sunday	Finish any last minute changes/updates/formatting, etc. and upload completed project.

Cryptography

Key Words: Enciphering, Deciphering, Modular Arithmetic, Linear Transformations, Hill n-cipher, digraph.

References to check: Look for books on mathematical approaches to cryptography. And, you can find information about modular arithmetic in computer programming texts or number theory texts.

Problems to code and write up:

In the following two problems, assume the use of the usual 26-letter alphabet with A=0 and Z=25, unless otherwise specified.

1. You intercept the message "SONAFQCHMWPTVEVY", which you know was enciphered using a Hill 2-cipher. An earlier statistical analysis of a long string of intercepted cipher-text revealed that the most frequently occurring cipher-text digraphs were "KH" and "XW" in that order. You take a guess that those digraphs correspond to "TH" and "HE", respectively, since those are the most frequently occurring digraphs in most long plaintext messages on the subject you think is being discussed. Find the deciphering matrix and state was the message is supposed to be.

2. In order to increase the difficulty of breaking your crypto-system, you decide to encipher your messages using a Hill 2-cipher by first applying the matrix $\begin{bmatrix} 3 & 11 \\ 4 & 15 \end{bmatrix}$ working modulo 26 and then applying the matrix $\begin{bmatrix} 10 & 15 \\ 5 & 9 \end{bmatrix}$ working modulo 29. Thus, while your plain-texts are in the usual 26 letter alphabet, your cipher-texts will be in the alphabet with 0-25 as usual and blank=26, ?=27, and !=28.

a) Encipher the message “THIS IS A FUN PROJECT”.

b) Describe how to decipher a cipher-text by applying two matrices in succession, and decipher “ZMOY”.

c) Under what conditions is a matrix with entries module 29 invertible modulo 29?

3. Create and solve one cryptography problem on your own.

Linear Programming

Key Words: Max-min problems, Feasible solution/region, Optimal solution, Decision variables, Objective function, Constraints, Simplex method, Standard Form, Slack variables.

References to check: Basic books on Operations Research (a branch of applied mathematics which studies these types of problems) are a good place to look for information.

Problems to code and write up:

1. Solve the following linear programming problem using the simplex method:

Minimize: $z=20x_1+10x_2+15x_3$

subject to

$$3x_1+2x_2+5x_3\leq 55$$
$$2x_1+x_2+x_3\leq 26$$
$$x_1+x_2+3x_3\leq 30$$
$$5x_1+2x_2+4x_3\leq 57$$
$$x_{1,}x_{2,}x_3\geq 0$$

2. Seven patients require blood transfusions. We assume four blood types: A, AB, B, and O. Type AB is called the universal recipient; type O is called the universal donor. The blood supply and patient data is as follows:

Type	Supply	Cost per pint
A	7 pints	$1
AB	6 pints	$2
B	4 pints	$4
O	5 pints	$5

Patient	Blood Type	Need (pints)
1	A	2
2	AB	3
3	B	1
4	O	2
5	A	3
6	B	2
7	AB	1

The problem is to ensure that each patient receives the required amount of the proper type of blood and to use the existing supply so that the cost of replacement is minimized. Formulate this as a linear programming problem. Label the decision variables, objective function, and constraints. Then solve it using the simplex method.

3. Create and solve one linear programming problem on your own.

Markov Chains

Key Words: Stochastic process, Markov chains, Transition probabilities, State vectors, Steady-state vectors.

References to check: Markov Chains are often mentioned in books about probability or stochastic processes.

Problems to code and write up:

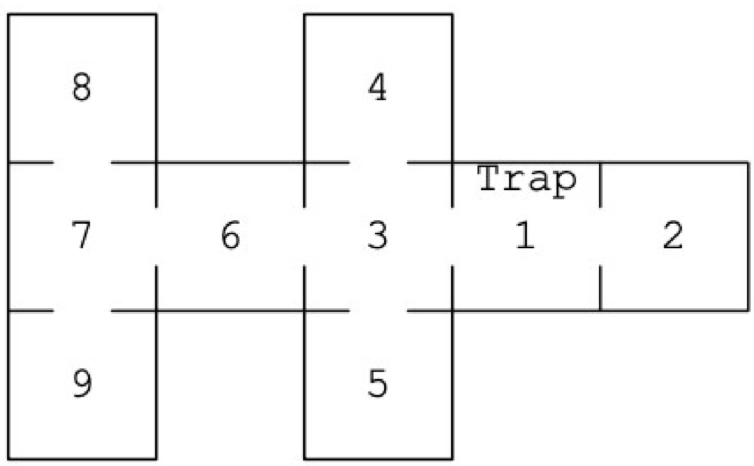
1. For the transition matrix $P = \begin{bmatrix} 0.4 & 0.5 \\ 0.6 & 0.5 \end{bmatrix}$,

   (a) calculate the first 10 state vectors if the initial state vector is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

   (b) Find the steady-state vector of P.

2. Puppy Gauss is either happy or pouting (a simple soul, our Gauss). If he is happy one day, he is happy the next day four times out of five. If he is pouting one day, the chances that he will also pout the next day are one time out of three. Over the long term, what are the chances that John is happy on any given day?

3. A mouse trap is placed in room 1 of the house with the pictured floor plan. Each time the mouse comes into room 1, he is trapped with probability p = 0.1. If he is not trapped, he leaves each room by one of its exits, chosen at random. Model the path of the mouse through the house as a Markov chain.



3. Create and solve one Markov chain problem on your own.

Game Theory

Key Words: Two person zero-sum game, Payoff matrix, Strategy, Expected payoff, Optimal strategy, Value of a game, Saddle point, Fundamental theorem of a 2-person zero-sum game.

References to check: Basic books on Operations Research (a branch of applied mathematics which studies these types of problems) or Game Theory (either from economics or from math) are a good place to look for information

Problems to code and write up:

1. Suppose that a game pays off according to the following table:

		B			
		i	ii	iii	iv
	i	-4	6	-4	1
A	ii	5	-7	3	8
	iii	-8	0	6	-2

(a) Suppose that player A uses strategy i half of the time, strategy iii half of the time, and strategy ii none of the time. Suppose also that player B uses each of the four strategies one fourth of the time. Find the expected payoff of the game.

(b) If player B keeps his strategy the same as in part (a), what strategy should player A choose to maximize her expected payoff?

(c) If player A keeps her strategy the same as in part (a), what strategy should player B choose to maximize his expected payoff?

2. Two clothing stores in a shopping center compete for the weekend trade. On a clear day the larger store gets 60% of the business and on a rainy day the larger store gets 80% of the business. Either or both stores may hold a sidewalk sale on a given weekend, but the decision must be made a week in advance and in ignorance of the competitor's plans. If both have a sidewalk sale, each gets 50% of the business. If, however, one holds the sale and the other doesn't, the one conducting the sale gets 90% of the business on a clear day and 10% on a rainy day. It rains 40% of the time. How frequently should each retailer conduct sales?

3. Create and solve one game theory problem on your own.

Population Growth

Key Words: Leslie model of population growth, Eigenvalues and eigenvectors, diagonalization of matrices, Positive and negative eigenvalues, Net reproduction rate.

References to check: Try books on demography or population growth models, especially those that mention Leslie models.

Problems to code and write up:

1. Suppose a certain animal population is divided into two age classes and has a Leslie matrix $L = \begin{bmatrix} 1 & 1.5 \\ 0.5 & 0 \end{bmatrix}$.

   (a) Calculate the positive eigenvalue of L and the corresponding eigenvector.

   (b) Beginning with the initial age distribution vector $x = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$ calculate the age distribution after 1, 2, 3, 4, and 5 years.

   (c) Calculate the age distribution after 6 years in two ways: once by using L, and once by using the eigenvalue you found in (a).

2. Show that the net reproduction rate of these animals can be interpreted as the average number of daughters born to a single female during her expected lifetime. Using this interpretation, reason that a population is eventually decreasing if and only if its net reproduction rate is less than one.

3. Suppose the oldest age attained by the females of a certain breed of dog is 15 years. Divide the population into three age classes of length 5 years: 0-5 years, 5-10 years, 10-15 years. Suppose that dogs under the age of 5 have 3 puppies on average per year, dogs between 5 and 10 years have 4 puppies on average per year, and dogs between 10 and 15 years have no puppies. Further, suppose that nine-tenths of dogs under the age of 5 are expected to live to the next year, while only three-fourths of dogs between the ages of 5 and 10 years are expected to live to the next year.

   (a) Find the Leslie matrix for this population.

   (b) If there are originally 1000 female dogs in each age class, find the population after 2 years. Find the population after 20 years.

   (c) Find the net reproductive rate.