Math1050 Final Exam
Spring, 2009

Name ______________________________

Instructions:

- Show all work as partial credit will be given where appropriate.
- If no work is shown, there may be no credit given.
- All final answers should be written in the space provided on the exam and in simplified form.
- No calculators allowed!

DO *NOT* WRITE IN THIS TABLE!!!
(It is for grading purposes.)

Grade:

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
EC	

Raw Total (out of 160 points)

Total (percentage)

1) (25 pts) If $f(x)=\sqrt{2x-4}$, $g(x)=\dfrac{3}{5x-1}$, and $h(x)=(x-2)^3+1$ find each of the following, simplifying as far as possible:

(a) Domain of f(x) and g(x).

$$2x-4 \geq 0 \iff 2x \geq 4 \iff x \geq 2$$

Domain of f(x): $x \geq 2$

$$5x-1 \neq 0 \iff x \neq \tfrac{1}{5}$$

Domain of g(x): $x \in \mathbb{R},\ x \neq \frac{1}{5}$

(b) $f(3)$

$$f(3) = \sqrt{2(3)-4} = \sqrt{6-4} = \sqrt{2}$$

$f(3)$ = $\sqrt{2}$

(c) $(f \circ g)(x)$

$$f(g(x)) = f\left(\frac{3}{5x-1}\right) = \sqrt{2\left(\frac{3}{5x-1}\right)-4}$$

$$= \sqrt{\frac{6}{5x-1}-4}$$

$(f \circ g)(x)$ = $\sqrt{\dfrac{6}{5x-1}-4}$

(Note: This is #1 continued!)

$f(x)=\sqrt{2x-4}$, $g(x)=\dfrac{3}{5x-1}$, **and** $h(x)=(x-2)^3+1$

(d) $g(a+b)$

$$g(a+b)=\frac{3}{5(a+b)-1} = \frac{3}{5a+5b-1}$$

$g(a+b)$ = $\dfrac{3}{5a+5b-1}$

(e) $h^{-1}(x)$

$$y=(x-2)^3+1$$

$$x=(y-2)^3+1$$

$$x-1=(y-2)^3$$

$$\sqrt[3]{x-1}=y-2$$

$$\sqrt[3]{x-1}+2=y$$

$$h^{-1}(x)=\sqrt[3]{x-1}+2$$

OR "pants technique"

$h(x)=(x-2)^3+1$

① -2
② $\wedge 3$
③ $+1$

$$h^{-1}(x)=\sqrt[3]{x-1}+2$$

$h^{-1}(x)$ = $\sqrt[3]{x-1}+2$

2) (10 pts) Solve the inequality.

$$\frac{2}{x-3} \geq \frac{1}{x+2} \iff \frac{2}{x-3} - \frac{1}{x+2} \geq 0$$

$$\frac{2(x+2) - 1(x-3)}{(x-3)(x+2)} \geq 0$$

$$\frac{2x+4-x+3}{(x-3)(x+2)} \geq 0 \iff \frac{x+7}{(x-3)(x+2)} \geq 0$$

critical values: $x = -7, 3, -2$

sign line: − + − +

$-7 \quad -2 \quad 3$

① ② ③ ④

test $\quad \frac{x+7}{(x-3)(x+2)}$

① $x = -10$ $\quad \frac{-}{-(-)} \to -$

② $x = -5$ $\quad \frac{+}{-(-)} \to +$

③ $x = 0$ $\quad \frac{+}{-(+)} \to -$

④ $x = 4$ $\quad \frac{+}{+(+)} \to +$

we want all regions that are positive (+) or zero and the expression is zero when $x = -7$

Solution: $[-7, -2) \cup (3, \infty)$

or $-7 \leq x < -2$ or $x > 3$

3) (15 pts) Use the questions below to help you find all the zeros of this polynomial function and write it as the product of linear factors. (Remember to show all your work!)

$f(x) = x^3 + 2x^2 - 4x - 8$

(a) How many possible positive roots are there? 1

(b) How many possible negative roots are there? 2 or 0

$f(-x) = -x^3 + 2x^2 + 4x - 8$

(c) List all the possible rational roots: $\pm 8, \pm 4, \pm 2, \pm 1$

(d) List all the zeros of this function, along with their multiplicity:

zero/root:	multiplicity:
-2	2
2	1

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -4 & -8 \\ & & -2 & 0 & +8 \\ \hline & 1 & 0 & -4 & 0 \end{array}$$

$\Rightarrow f(x) = (x+2)(x^2-4)$

$= (x+2)(x-2)(x+2)$

$= (x+2)^2(x-2)$

(e) Write the polynomial as a product of linear factors:

$f(x) = (x+2)^2(x-2)$

4) (20 pts) For this rational function, answer the following questions.

$$f(x)=\frac{(x-1)(x+3)}{(x+2)(x-5)}$$

(a) What is its domain? $x \in \mathbb{R},\ x \neq -2,\ x \neq 5$

(b) Vertical Asymptote(s) (if any): $x=-2,\ x=5$

(c) Horizontal Asymptote(s) (if any): $y=1$

$$f(x)=\frac{x^2+2x-3}{x^2-3x-10} \sim \frac{x^2}{x^2}=1 \Rightarrow y=1$$ as x gets huge

(d) y-intercept(s) (if any): $(0, 3/10)$

$$f(0)=\frac{(0-1)(0+3)}{(0+2)(0-5)}=\frac{-3}{-10}=\frac{3}{10}$$

(e) x-intercept(s) (if any): $(1,0)$ and $(-3,0)$

$$0=\frac{(x-1)(x+3)}{(x+2)(x-5)} \Leftrightarrow 0=(x-1)(x+3)$$
$$x=1,-3$$

(f) Sketch the graph.

$(6, 5\frac{5}{8})$

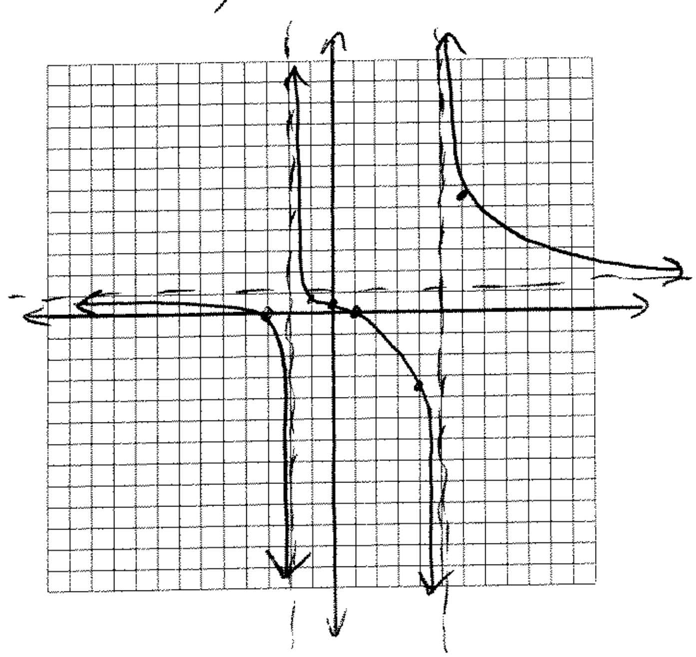
$$f(6)=\frac{5(9)}{8(1)}=\frac{45}{8}=5\frac{5}{8}$$

$(4, -3\frac{1}{2})$

$$f(4)=\frac{3(7)}{6(-1)}=\frac{-7}{2}$$

$(-1, 2/3)$

$$f(-1)=\frac{-2(2)}{1(-6)}=\frac{2}{3}$$



5. (20 pts) Solve each equation. (Beware of domain restrictions.)

(a) $56=8e^{2x-3}$

$7 = e^{2x-3}$

$\ln 7 = 2x-3$

$3 + \ln 7 = 2x$

$x = \frac{3+\ln 7}{2}$

x = $\frac{3+\ln 7}{2}$

(b) $\log_2(x+1)+\log_2(x-1)=3$

$\log_2[(x+1)(x-1)] = 3$

$2^3 = (x+1)(x-1)$

$8 = x^2-1$

$0 = x^2-9$

$x = 3, -3$

but if $x=-3$, we get log of negative #

$\Rightarrow x \neq -3$

x = 3

6) (30 pts) Given the matrices A, B, C and D, compute the following, if possible. If it's not possible, state the reason why.

$$A=\begin{bmatrix}1 & -1 & 0\\ 1 & 0 & -1\\ 6 & -2 & -3\end{bmatrix};\quad B=\begin{bmatrix}3 & 4\\ -2 & 1\end{bmatrix},\quad C=\begin{bmatrix}22\\ 0\end{bmatrix} \text{ and } D=\begin{bmatrix}2 & 5 & 1\\ 0 & -3 & -1\end{bmatrix}.$$

(a) DB

$$\begin{bmatrix}2 & 5 & 1\\ 0 & -3 & -1\end{bmatrix}\begin{bmatrix}3 & 4\\ -2 & 1\end{bmatrix} \times$$

we can't do this multiplication because # cols of D ≠ # rows of B.

$(2\times 3)(2\times 2)$

DB = not possible

(b) A^{-1}

$(-6)\ (-1)$

$$\left[\begin{array}{ccc|ccc}1 & -1 & 0 & 1 & 0 & 0\\ 1 & 0 & -1 & 0 & 1 & 0\\ 6 & -2 & -3 & 0 & 0 & 1\end{array}\right] \quad (-4)\left[\begin{array}{ccc|ccc}1 & -1 & 0 & 1 & 0 & 0\\ 0 & 1 & -1 & -1 & 1 & 0\\ 0 & 4 & -3 & -6 & 0 & 1\end{array}\right]$$

$$\left[\begin{array}{ccc|ccc}1 & -1 & 0 & 1 & 0 & 0\\ 0 & 1 & -1 & -1 & 1 & 0\\ 0 & 0 & 1 & -2 & -4 & 1\end{array}\right] \quad \left[\begin{array}{ccc|ccc}1 & -1 & 0 & 1 & 0 & 0\\ 0 & 1 & 0 & -3 & -3 & 1\\ 0 & 0 & 1 & -2 & -4 & 1\end{array}\right]$$

$$\left[\begin{array}{ccc|ccc}1 & 0 & 0 & -2 & -3 & 1\\ 0 & 1 & 0 & -3 & -3 & 1\\ 0 & 0 & 1 & -2 & -4 & 1\end{array}\right]$$

$$A^{-1} = \begin{bmatrix}-2 & -3 & 1\\ -3 & -3 & 1\\ -2 & -4 & 1\end{bmatrix}$$

(Note: This is #6 continued.)

$$A=\begin{bmatrix}1 & -1 & 0\\ 1 & 0 & -1\\ 6 & -2 & -3\end{bmatrix},\quad B=\begin{bmatrix}3 & 4\\ -2 & 1\end{bmatrix},\quad C=\begin{bmatrix}22\\ 0\end{bmatrix} \text{ and } D=\begin{bmatrix}2 & 5 & 1\\ 0 & -3 & -1\end{bmatrix}.$$

(c) BD

$$BD = \begin{bmatrix}3 & 4\\ -2 & 1\end{bmatrix}\begin{bmatrix}2 & 5 & 1\\ 0 & -3 & -1\end{bmatrix}$$

$(2\times 2)(2\times 3)$

$\to 2\times 3$

$$= \begin{bmatrix}6+0 & 15-12 & 3-4\\ -4+0 & -10-3 & -2-1\end{bmatrix}$$

BD = $\begin{bmatrix}6 & 3 & -1\\ -4 & -13 & -3\end{bmatrix}$

(d) $|A|$

$$\begin{array}{ccc} + & - & + \end{array}$$

$$\begin{vmatrix}1 & -1 & 0\\ 1 & 0 & -1\\ 6 & -2 & -3\end{vmatrix} = 1\begin{vmatrix}0 & -1\\ -2 & -3\end{vmatrix} - (-1)\begin{vmatrix}1 & -1\\ 6 & -3\end{vmatrix} + 0$$

$$= 1(0-2) + 1(-3-(-6))$$

$$= -2 + 3 = 1$$

$|A|$ = 1

(Note: This is #6 continued.)

$$A=\begin{bmatrix}1 & -1 & 0\\ 1 & 0 & -1\\ 6 & -2 & -3\end{bmatrix},\quad B=\begin{bmatrix}3 & 4\\ -2 & 1\end{bmatrix},\quad C=\begin{bmatrix}22\\ 0\end{bmatrix} \text{ and } D=\begin{bmatrix}2 & 5 & 1\\ 0 & -3 & -1\end{bmatrix}.$$

(e) Let $X=\begin{bmatrix}x\\ y\end{bmatrix}$. Set up $BX=C$ (using B and C as given) to represent a linear system of equations.

$$\begin{bmatrix}3 & 4\\ -2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}22\\ 0\end{bmatrix}$$

or

$$3x+4y=22$$
$$-2x+y=0$$

(f) Solve $BX=C$ for the values of x and y.

I'll use substitution.

$$y=2x \Rightarrow 3x+4(2x)=22$$
$$3x+8x=22$$
$$11x=22$$
$$x=2$$

$$\Rightarrow y=2(2)=4$$

$(x, y) =$ $(2, 4)$

7) (10 pts) Solve this system of equations.

$$x^2 - 3y = 10$$
$$3(\ x + y = 6\ )$$

$$x^2 - 3y = 10$$
$$+\quad 3x + 3y = 18$$
$$x^2 + 3x = 28$$
$$x^2 + 3x - 28 = 0$$
$$(x-4)(x+7) = 0$$
$$x = 4, -7$$

if $x = 4$, $y = ?$

$4 + y = 6 \Rightarrow y = 2$

if $x = -7$, $y = ?$

$-7 + y = 6$

$y = 13$

$(x, y) =$ $(4, 2)\quad(-7, 13)$

8) (10 pts) For the sequence $a_n = \frac{(-1)^{n+1}}{2n+5}$, answer the following questions.

(a) Write the first five terms.

n	a_n
1	$1/7$
2	$-1/9$
3	$1/11$
4	$-1/13$
5	$1/15$

$$a_1 = \frac{(-1)^2}{2(1)+5} = \frac{1}{7}$$
$$a_2 = \frac{(-1)^3}{4+5} = \frac{-1}{9}$$
$$a_3 = \frac{(-1)^4}{2(3)+5} = \frac{1}{11}$$
$$a_4 = \frac{(-1)^5}{2(4)+5} = \frac{-1}{13}$$

(b) Is this sequence arithmetic geometric or neither ? (circle one)

9) (10 pts) For the geometric sequence

2, 6, 18, 54, . . .

(a) Write the iterative formula for a_n. (Assume that the first term given is the a_1 term.)

common ratio $r = 3$, $a_1 = 2$

$a_n = 2(3^{n-1})$ or $\frac{2}{3}(3^n)$

a_n = $2(3^{n-1})$

(b) What is the sum of the first 8 terms?

$$\sum_{n=1}^{8} 2(3^{n-1}) = \frac{2(1-3^8)}{1-3} = \frac{2(1-9^4)}{-2} = \frac{\cancel{2}(1-81^2)}{-\cancel{2}}$$

$$= -(1-81^2) = -(1-6561)$$

$$= 6560$$

$$\begin{array}{r} 81 \\ \times 81 \\ \hline 81 \\ 648 \\ \hline 6561 \end{array}$$

Sum = 6560

10) (10 pts) Expand and simplify, using the Binomial Theorem.

$(x-2y)^5$

$$\begin{array}{c} 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \\ 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \end{array}$$

$$= 1(x^5) + 5(x^4)(-2y)^1 + 10(x^3)(-2y)^2$$

$$+ 10(x^2)(-2y)^3 + 5(x^1)(-2y)^4$$

$$+ 1(-2y)^5$$

$$= x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 80xy^4 - 32y^5$$

$(x-2y)^5$ = $x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 80xy^4 - 32y^5$

Extra Credit: (8 pts) Use mathematical induction to prove that
$$1+2+2^2+2^3+2^4+\ldots+2^{n-1}=2^n-1 \quad .$$

Pf ① Check for $n=1$.

if $n=1$, $2^{n-1}=2^{1-1}=2^0=1$

$\Rightarrow$ left side is 1

right side is $2^1-1=2-1=1$ $\quad 1=1$ ✓

② Assume it's true for $n=1,2,\ldots,k$.

Check for $n=k+1$.

$$\underbrace{1+2+2^2+2^3+\ldots+2^{k-1}}_{=2^k-1}+2^{(k+1)-1}$$

$$=2^k-1+2^k = 2(2^k)-1 = 2^{k+1}-1 \quad ✓$$