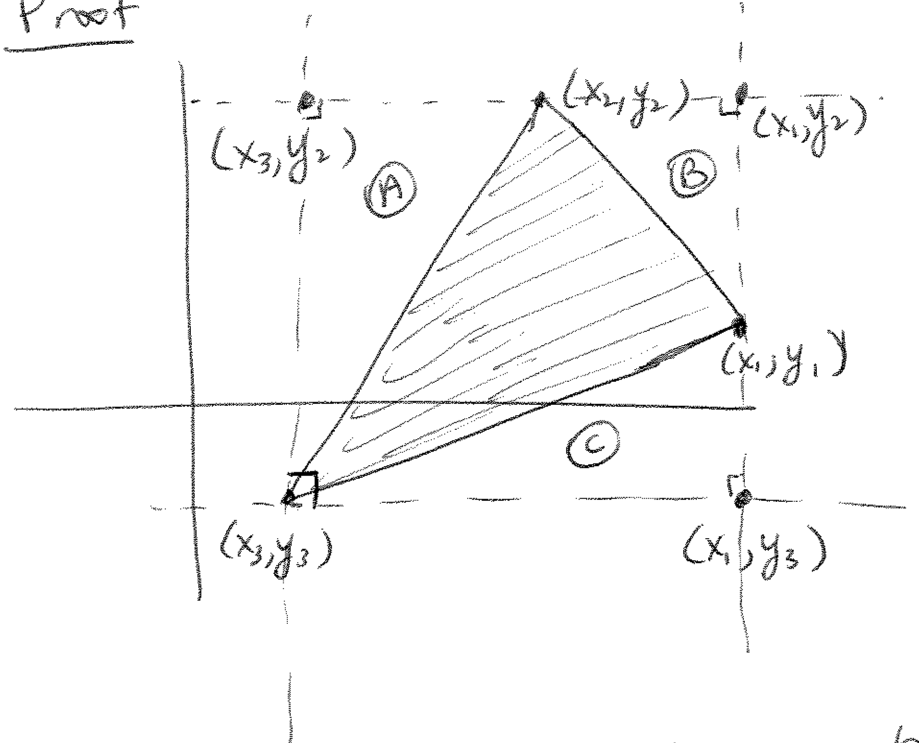
Claim Area of triangle defined by vertices (x_1, y_1) (x_2, y_2) and (x_3, y_3) is

$$A = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}.$$

Proof



Consider this triangle. Form (dotted) rectangle that contains triangle, and label points of rectangle's vertices.

Then area of triangle can be calculated by

$$A_\Delta = A_\square - A_A - A_B + A_C$$

(i.e. area of the rectangle minus areas of triangles A, B and C).

We'll now use the formulas $A = lw$ for area of a rectangle and $A = \frac{1}{2}bh$ for each of the right triangles.

$$\Rightarrow A_{\Delta} = \overbrace{(x_1 - x_3)(y_2 - y_3)}^{A_{\square}} - \overbrace{\tfrac{1}{2}(x_2 - x_3)(y_2 - y_3)}^{A_{Ⓐ}}$$

$$- \underbrace{\tfrac{1}{2}(x_1 - x_2)(y_2 - y_1)}_{A_{Ⓑ}} - \underbrace{\tfrac{1}{2}(x_1 - x_3)(y_1 - y_3)}_{A_{Ⓒ}}$$

$$\Leftrightarrow A_{\Delta} = x_1 y_2 - x_1 y_3 - x_3 y_2 + \cancel{x_3 y_3} - \tfrac{1}{2}\cancel{x_2} y_2 + \tfrac{1}{2} x_2 y_3 + \tfrac{1}{2} x_3 y_2$$

$$\cancel{- \tfrac{1}{2} x_3 y_3} - \tfrac{1}{2} x_1 y_2 + \cancel{\tfrac{1}{2} x_1 y_1} + \tfrac{1}{2}\cancel{x_2} y_2 - \tfrac{1}{2} x_2 y_1$$

$$\cancel{- \tfrac{1}{2} x_1 y_1} + \tfrac{1}{2} x_1 y_3 + \tfrac{1}{2} x_3 y_1 \cancel{- \tfrac{1}{2} x_3 y_3}$$

$$= \underline{x_1 y_2} - \underline{x_1 y_3} - \underline{x_3 y_2} + \tfrac{1}{2} x_2 y_3 + \underline{\tfrac{1}{2} x_3 y_2} \underline{- \tfrac{1}{2} x_1 y_2}$$

$$- \tfrac{1}{2} x_2 y_1 + \underline{\tfrac{1}{2} x_1 y_3} + \tfrac{1}{2} x_3 y_1$$

$$= \tfrac{1}{2} x_1 y_2 - \tfrac{1}{2} x_1 y_3 - \tfrac{1}{2} x_2 y_1 + \tfrac{1}{2} x_2 y_3 + \tfrac{1}{2} x_3 y_1 - \tfrac{1}{2} x_3 y_2$$

$$= \tfrac{1}{2}\left(x_1 y_2 - x_1 y_3 - x_2 y_1 + x_2 y_3 + x_3 y_1 - x_3 y_2\right)$$

$$= \tfrac{1}{2}\left(\left(x_1 y_2 - x_2 y_1\right) - \left(x_1 y_3 - x_3 y_1\right) + \left(x_2 y_3 - x_3 y_2\right)\right)$$

$$= \tfrac{1}{2}\left(\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} - \begin{vmatrix} x_1 & y_1 \\ x_3 & y_3 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} \right)$$

$$= \tfrac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

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