3.1 Exponential Fns and Their Graphs

Defn

Exponential fn w/ base a denoted by

$$f(x)=a^x, \quad a>0, \quad a\neq 1, \quad x\in\mathbb{R}$$

Natural exponential base $= e$

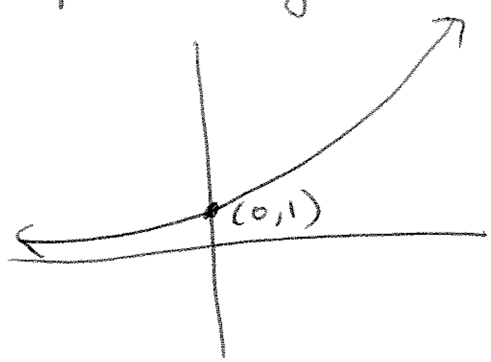
$e \in$ irrational #s, $e \simeq 2.71828182 8\ldots$

Note

① Power fn
$f(x)=x^a$
(a constant)
(variable = base)

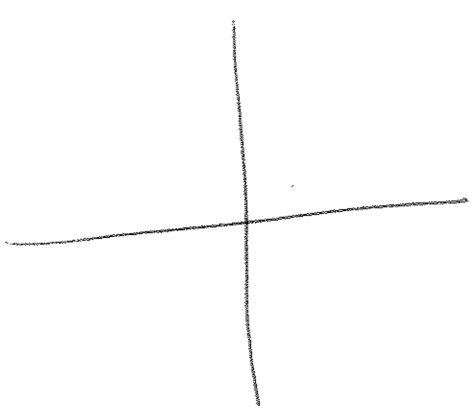
② Exponential fn
$f(x)=a^x$
(a constant)
(variable = exponent)

Graph of $y=e^x$

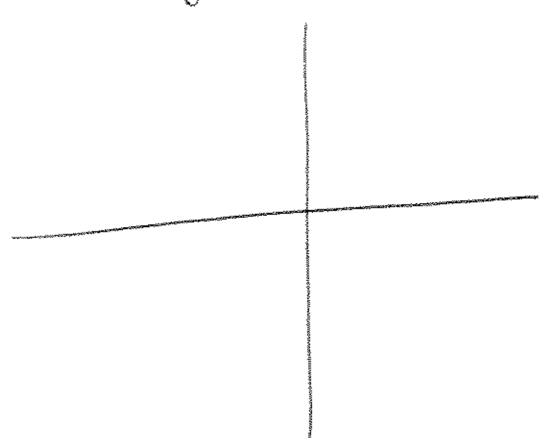


Ex 1 Sketch graph of $f(x)=4^{x-1}+2$
(notice transformations)

$y=4^x$



$y=4^{x-1}+2$



(☆ note: add # 33, 35, 37 to hw assignment)

3.1 (cont)

Ex 2 Sketch graph for $f(x) = -3e^{x-2} - 4$

Ex 3 Solve $\left(\frac{1}{5}\right)^{x+1} = 125$

let annual interest rate $= r$

Compound Interest
(exponential growth)

# yrs	amt in acct
0	\$100
1	\$100 $(1+r)$
2	$[\$100(1+r)](1+r) = 100(1+r)^2$
3	$[100(1+r)^2](1+r) = 100(1+r)^3$
$\vdots$	$\vdots$
n	$100(1+r)^n$

Simple Interest
(linear growth)

# yrs	amt in acct
0	\$100
1	$100 + 100r = 100(1+r)$
2	$100(1+r) + 100r = 100(1+2r)$
3	$100(1+2r) + 100r = 100(1+3r)$
$\vdots$	
n	$100(1+nr)$

3.1 (cont)

In general, the formula for compound interest is $\boxed{A = P\left(1+\frac{r}{n}\right)^{nt}}$

A = amt (or balance) after t years

P = principal

r = annual interest rate

t = # yrs

n = # compoundings per year

If we $\boxed{\text{compound continuously}}$, this turns into $\boxed{A = Pe^{rt}}$. (Why? you might ask... we'll be able to prove this to you in Calculus 2! :))

Ex 4 A deposit of \$5,000 is made in a trust fund that pays 7.5% interest, compounded continuously. After 50 years, how much will this fund be worth?

3.2 Logarithmic Fns and Their Graphs

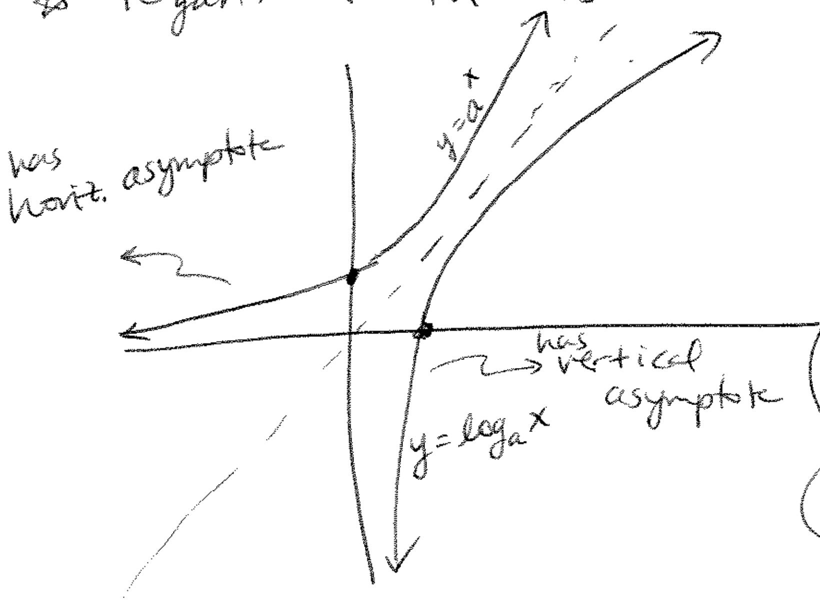
Defn

$\boxed{y = \log_a x \iff x = a^y}$ $\boxed{\text{domain } x > 0}$, $a > 0$, $a \neq 1$

$f(x) = \log_a x$ is a logarithmic fn w/ base a

read "log base a of x"

☆ logarithm fn is inverse of exponential fn



$\log_a(a^x) = x$

$= a^{\log_a x}$

Natural logarithm

occurs when $a = e$

$\Rightarrow f(x) = y = \log_e x = \ln x$

$x > 0$

Basic Logarithm Properties

① $\log_a 1 = 0$ (because $a^0 = 1$) $\forall a \neq 0$

② $\log_a a = 1$ ($\iff a^1 = a$)

③ $\log_a a^x = x$ ($\iff a^x = a^x$)

④ If $\log_a x = \log_a y$, then $x = y$. (because fn is invertible)

3.2 (cont)

Ex 1 Rewrite in equivalent form.

(a) $\log_8 4 = 2/3$

(b) $9^{3/2} = 27$

(c) $4^{-3} = \frac{1}{64}$

(d) $\log \frac{1}{1000} = -3$

Ex 2 Evaluate.

(a) $\log_a a^2$

(b) $\log_2 16$

(c) $8^{\log_8 7}$

(d) $\log_{2/3} 1$

Ex 3 Find domain, x-intercept & VA and sketch graph for $f(x) = \log_5(x-1) + 4$.

Ex 4 Find domain, x-intercept, VA and sketch graph for $f(x) = 3\ln x - 1$.

Ex 5 Solve $\ln(x-4) = \ln 2$

3.3 Properties of logarithms

Log Properties

1. $\log_a(xy) = \log_a x + \log_a y$
2. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
3. $\log_a x^m = m \log_a x$

Change of Base

$$\boxed{\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}}$$

Proof let $(a > 0)$

$$y = \log_a x$$

$$\Leftrightarrow \quad a^y = x$$

$$\Rightarrow \quad \ln a^y = \ln x$$

$$y(\ln a) = \ln x$$

$$y = \frac{\ln x}{\ln a}$$

Ex 1 Rewrite (using change of base) $\log_{7.1} x$

Ex 2 Evaluate (on your calculator) $\log_3 9.2$

Ex 3 Rewrite + simplify $\ln\left(\frac{6}{e^2}\right)$

3.3 (cont)

Ex 4 Evaluate (w/o calculator).

(a) $\log_3 81^{-0.2}$

(b) $\log_4 2 + \log_4 32$

(c) $\ln(\sqrt[4]{e^3})$

Ex 5 Use log properties to expand.

(a) $\ln\sqrt{x^2(x+2)}$

(b) $\log\left(\frac{x^2-1}{x^3}\right)$ $(x>1)$

Ex 6 Condense into one term.

(a) $3\log_3 x + 4\log_3 y - 5\log_3 z$

(b) $\frac{1}{2}\left[\log_4(x+1) + 2\log_4(x-1)\right] + 6\log_4 x$

Ex 7 Are these equivalent?

① $\log_7\sqrt{70}$ ② $\log_7 35$ ③ $\frac{1}{2} + \log_7\sqrt{10}$

3.4 Exponential and Logarithmic Eqns

Strategy

Exponential Eqn

① move terms around to isolate exponential on one side of eqn.

② (a) Rewrite as equivalent log eqn.

OR (b) take ln of both sides of eqn.

③ Finish solving.

Logarithmic Eqn

① Use log properties to condense all logs into single log expression (on one side).

② (a) exponentiate both sides (w/ base matching log base)

OR (b) rewrite as equivalent exponential eqn

③ Finish solving.

★ always check answers

Ex 1 Solve.

(a) $\left(\frac{1}{4}\right)^x = 64$

(b) $\ln x - \ln(x+1) = 2$

3.4 (cont)

Ex 2 Solve.

(a) $\log 4x - \log(12+\sqrt{x}) = 2$

(b) $-14 + 3e^{x} = 11$

(c) $\left(16 - \frac{0.878}{26}\right)^{3x} = 30$

3.4 (cont)

Ex 2 (cont)

(d) $$\frac{119}{e^{6x}-14}=7$$

(e) $\ln(x+1)-\ln(x-2)=\ln x$