7.2 Ex 3 (from notes)

$$\int \frac{\ln x}{\sqrt{x}}\,dx$$ (do Integration by Parts)

$u = \ln x$ $\qquad v = 2\sqrt{x}$

$du = \frac{1}{x}\,dx$ $\qquad dv = \frac{1}{\sqrt{x}}\,dx = x^{-1/2}\,dx$

$$= 2\sqrt{x}\ln x - 2\int \frac{\sqrt{x}}{x}\,dx$$

$$= 2\sqrt{x}\ln x - 2\int x^{-1/2}\,dx$$

$$= 2\sqrt{x}\ln x - 2(2\sqrt{x}) + C$$

$$= 2\sqrt{x}\ln x - 4\sqrt{x} + C$$

Ex 6 $\boxed{n \neq 0, 1}$ $\quad \int \cos^n x\,dx = \int \cos x \cos^{n-1} x\,dx$

$u = \cos^{n-1} x$ $\qquad v = \sin x$

$du = (n-1)\cos^{n-2} x(-\sin x)\,dx$ $\qquad dv = \cos x\,dx$

$$= \sin x \cos^{n-1} x + (n-1)\int \sin^2 x \cos^{n-2} x\,dx$$

Remember $\quad \sin^2 x + \cos^2 x = 1 \iff \sin^2 x = 1 - \cos^2 x$

$$\int \cos^n x\,dx = \sin x \cos^{n-1} x + (n-1)\left[\int \cos^{n-2} x\,dx - \int \cos^n x\,dx\right]$$

$$\iff \int \cos^n x\,dx = \sin x \cos^{n-1} x + (n-1)\int \cos^{n-2} x\,dx - (n-1)\int \cos^n x\,dx$$

$$(1+(n-1))\int \cos^n x\,dx = \sin x \cos^{n-1} x + (n-1)\int \cos^{n-2} x\,dx$$

$$n\int \cos^n x\,dx = \sin x \cos^{n-1} x + (n-1)\int \cos^{n-2} x\,dx$$

$$\boxed{\int \cos^n x\,dx = \frac{1}{n}\left[\sin x \cos^{n-1} x + (n-1)\int \cos^{n-2} x\,dx\right]}$$

Notice that we don't get a final answer for our integral. However, we have at least reduced the integral to one of lower power of $\cos x$. Thus, if $n=4$, we'd need this technique twice to be finished.

Likewise,

$$\boxed{\int \sin^n x\,dx = \frac{1}{n}\left[-\cos x \sin^{n-1} x + \frac{n-1}{n}\int \sin^{n-2} x\,dx\right]}$$