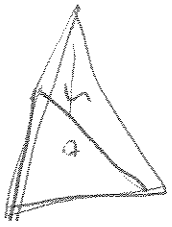
3d Geometry Jeopardy

Polyhedra

10 points-- Why can't we make a Platonic solid with hexagonal faces?

because 3 regular hexagons lie flat and cannot form a dihedral angle

20 points-- Draw a right triangular pyramid.

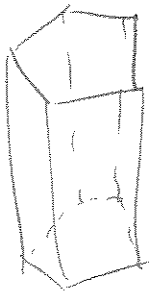


(a pyramid w/ equilateral △ base)

30 points-- List the Platonic Solids.

cube
dodecahedron
~~duel~~ tetrahedron
icosahedron
octahedron

40 points-- A prism has 96 edges. How many vertices and faces does it have?



$F + V - 2 = E$

$F + V - 2 = 96$

if n = # of sides in polygon base, then

$F = 2 + n$

and $V = 2n$

$\Rightarrow 2 + n + 2n - 2 = 96$

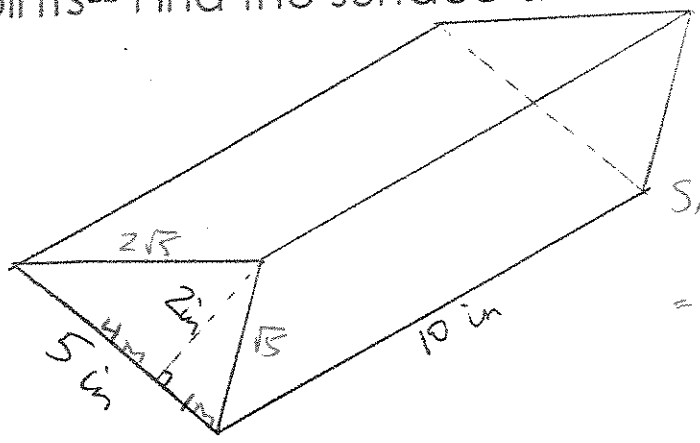
$3n = 96$

$n = 32 \Rightarrow F = 34$

$V = 64$

Surface Area

10 points-- Find the surface area of the following solid.



$A_{\triangle} = \frac{1}{2}(5)(2) = 5 \text{ in}^2$

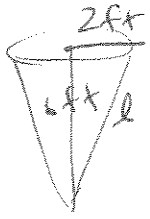
$SA = 2(5) + (\sqrt{5}+2\sqrt{5}+5)10$

$= 10(6+3\sqrt{5}) \text{ in}^2$

20 points-- Find the surface area of a sphere with radius of 5 meters.

$SA = 4\pi r^2 = 4\pi(25) = 100\pi \text{ m}^2$

30 points-- Find the surface area of a right circular cone with radius of 2 ft. and height of 6 ft.



$SA = \pi(2^2) + \frac{1}{2}(2\pi(2))2\sqrt{10}$

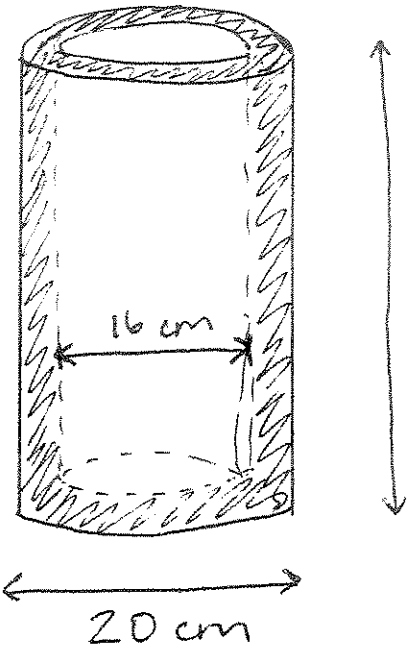
$= 4\pi + 4\sqrt{10}\pi = 4\pi(1+\sqrt{10}) \text{ ft}^2$

$2^2 + 6^2 = \ell^2$

$\ell = \sqrt{40} = 2\sqrt{10}$

Surface Area (continued)

40 points-- Find the surface area of the following shell.



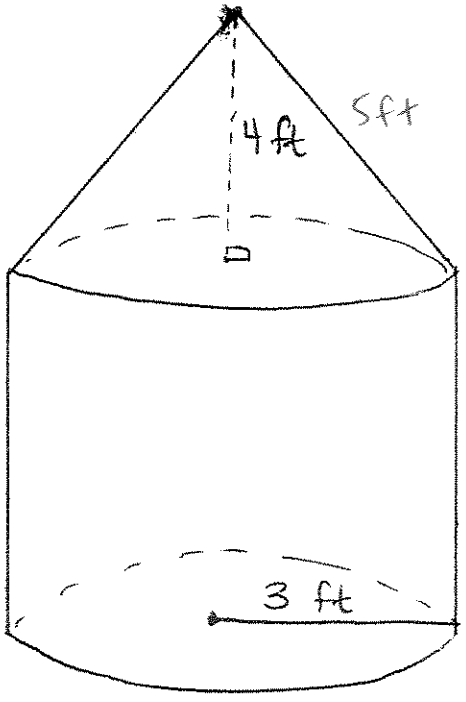
$A_{\text{base}} = \pi(10^2) - \pi(8^2)$
$= 36\pi$

$A_{\text{outer}} = 15(20\pi)$
$= 300\pi$

$A_{\text{inner}} = 15(16\pi)$
$= 240\pi$

$SA = 300\pi + 240\pi + 36\pi = 576\pi \text{ cm}^2$

50 points-- Find the surface area of the following solid.



$A = \pi(3)(5) = 15\pi$

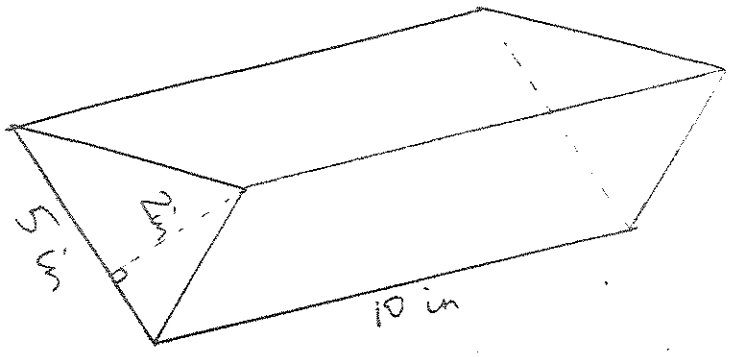
$A = 2\pi(3)(10) = 60\pi$

$A = \pi(3^2) = 9\pi$

$SA = 15\pi + 60\pi + 9\pi = 84\pi \text{ ft}^2$

Volume

10 points-- Find the volume of the following solid.



$V = Ah$

$= \left(\frac{1}{2}(5 \cdot 2)\right)10$

$= 50 \text{ in}^3$

20 points-- Find the volume of a sphere with radius of 5 meters.

$V = \frac{4}{3}\pi(5^3) = \frac{500\pi}{3} \text{ m}^3$

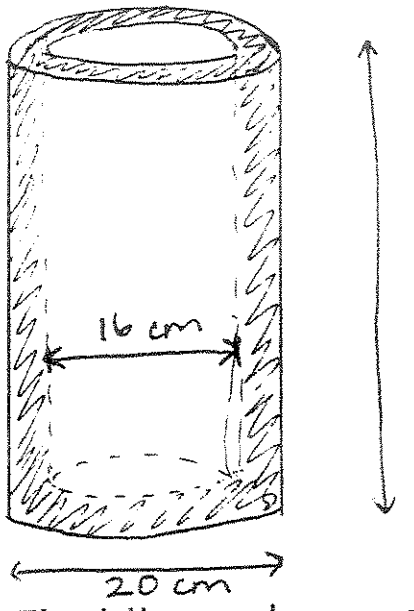
30 points-- Find the volume of a right circular cone with radius of 2 ft. and height of 6 ft.

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(2^2)(6)$

$= 8\pi \text{ ft}^3$

Volume (continued)

40 points-- Find the volume of the following shell.

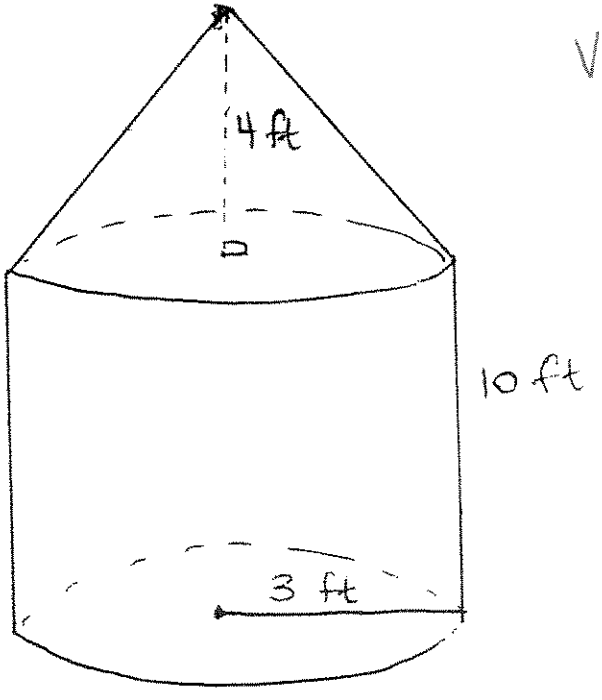


$V = V_{outer} - V_{inner}$

$= \pi(10^2)15 - \pi(8^2)(15)$

$= 540\pi \text{ cm}^3$

50 points-- Find the volume of the following solid.



$V = \frac{1}{3}\pi(3^2)(4) + \pi(3^2)(10)$

$= 12\pi + 90\pi$

$= 102\pi \text{ ft}^3$

Scaling

10 points-- If a cube's sides double in length, what happens to its surface area?

it's multiplied by $2^2 = 4$

20 points-- If a cube's sides triple in length, what happens to its volume?

it's multiplied by $3^3 = 27$

30 points-- We have a scale model prism whose height is 5 inches, and we want the actual prism to have a height of 10 feet. What is the relationship between the scale model's surface area and the actual surface area?

$5 \text{ in} \rightarrow 10 \text{ ft} = 5 \text{ m} \rightarrow 120 \text{ m}$

s = scaling factor

$5s = 120$

$s = 24$

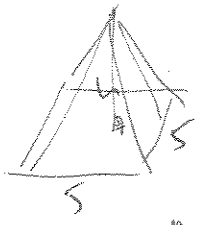
$\Rightarrow SA_{new} = 24^2 \, SA_{model}$

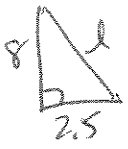
Scaling (continued)

40 points-- We have a scale model prism whose height is 5 inches, and we want the actual prism to have a height of 10 feet. What is the relationship between the scale model's volume and the actual volume?

$$V_{actual} = 24^3 V_{model}$$

50 points-- For a right square pyramid with height h = 8 inches and the base side length = 5 inches, what is the surface area and volume? If we scale that up to have a height of 3 ft, what is its surface area and volume?





$8^2 + 2.5^2 = \ell^2$

$\ell^2 = \frac{281}{4}$

$\ell = \frac{\sqrt{281}}{2}$

$SA = 5^2 + \frac{1}{2}(20)\left(\frac{\sqrt{281}}{2}\right)$

$= 25 + 5\sqrt{281}$

$\approx 108.8 \text{ in}^2$

$V = \frac{1}{3}Ah = \frac{1}{3}(25)(8) = \frac{200}{3}$

$= 66.\overline{6} \text{ in}^3$

Hodge Podge

10 points-- Give an exact definition of a sphere.

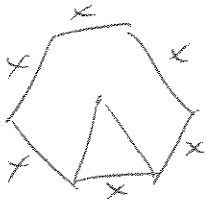
the set of points equidistant from a fixed pt (called the center) in 3d

20 points-- What is Euler's Formula and what does it apply to?

$F+V-2=E$

all polyhedra

30 points-- Find the area of a regular hexagon whose sides are 4 cm in length.



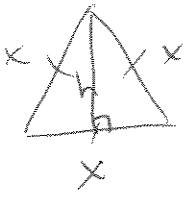
$A_0 = 6\left(\frac{\sqrt{3}}{4}x^2\right)$

$= \frac{3\sqrt{3}}{2}x^2$

$\Rightarrow A = \frac{3\sqrt{3}}{2}(4^2) =$

$24\sqrt{3}\ cm^2$

area of equilateral Δ:



$h^2+\left(\frac{1}{2}x\right)^2 = x^2$

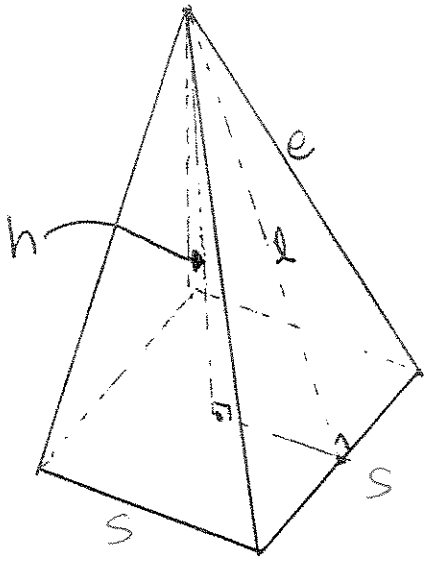
$h^2 = \frac{3}{4}x^2$

$h = \frac{\sqrt{3}}{2}x$

$\Rightarrow A_\Delta = \frac{1}{2}x\left(\frac{\sqrt{3}}{2}x\right)$

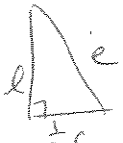
$= \frac{\sqrt{3}}{4}x^2$

Hodge Podge (continued)
40 points-- What is the relationship between h, l and e (as drawn on this pyramid)?



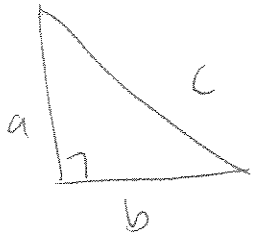
$h^2 + (\frac{1}{2}s)^2 = l^2$

and



$l^2 + (\frac{1}{2}s)^2 = e^2$

50 points-- State the Pythagorean Theorem and give a proof.



In a right triangle, the sum of the lengths of the legs squared is equal to the hypotenuse length squared.

$a^2 + b^2 = c^2$