Ages of Academy Award Winners 1928-2000

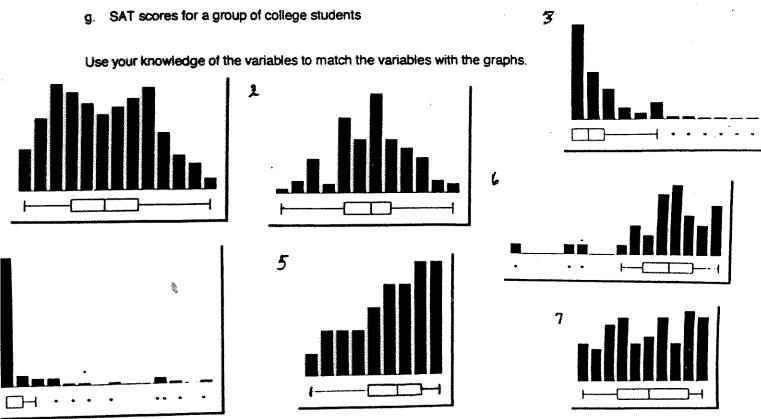
	Best Actors	Best Actresses
		- Carrier - Carr
	•	2 1 2 4 4 4 4 4
		• 5566666666777889999
	4433222110	3 0 0 0 0 1 1 2 3 3 3 4 4 4 4 4 4 4
5 5 5 5 7 7	7888888899	• 5556778889
4433333222	1111000000	4 0 1 1 1 1 1 1 2 2
	5887766655	• 55899
	43211	5
	6665	•
	0	6 0 1 1 2
	•	7 4
	. 6	• • • • • • • • • • • • • • • • • • •
		8 0
		•

Matching graphs to variables.

1. Match these variables to their graphs.

Consider the following list of variables and graphs:

- a. age at death of a sample of 34 people.
- b. The last digit in the social security number of 40 students.
- c. scores on a fairly easy examination in statistics
- d. number of months after going off the pill it took to get pregnant
- e. heights of a group of college students
- f. number of medals won by medal-winning countries in the 1992 Winter Olympics



2. After you have matched the graphs to the variables, describe each distribution and any unusual features (e.g. is the distribution skewed to the right? Is it symmetric? Are there outliers? Why might this be so?).

3. Name one variable which might have asymmetric distribution, one that has distribution skewed to the right, and one that has distribution skewed to the left.

4. In each case estimate whether the mean is greater than, less than, or equal to the median, and explain your reasoning.

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Use the scattergrams A-F to answer all of the questions on this page.

Match the following to the scattergram which best fits the description.

5. Perfect positive correlation (r=1)

6. Strong positive correlation (r = .91)

7. Weak positive correlation (r = .3)

______8. No correlation (r = 0)

_____9. Strong negative correlation (r = -.85)

_____10. Perfect negative correlation (r = -1)

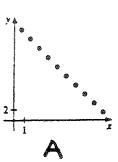
Match the following variables to a suitable scattergram.

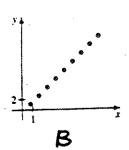
_____11. x = infant age in days, y = length in inches

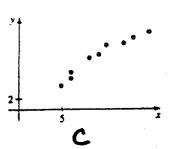
_____12. x = years smoked, y = years you will live

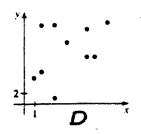
_____13. x = height in cm, y = GPA

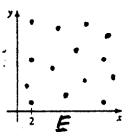
_____14. x = weight in pounds, y = weight in kilograms

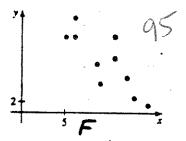












Math4020 10.2-1 Notes Measures of Central Tendency

Mode-->The number that occurs most frequently; there can be more than one mode; if each number appears equally often, then there is no mode at all.

(mode is great for shoe size, ice cream flavor, etc.)

- Median—>The middle number when the numbers are all listed in increasing (or decreasing) order. If there are an even number of numbers, then take the arithmetic average (a.k.a. Mean) of the two middle numbers.
- <u>Mean->(a.k.a.</u> Arithmetic average) The number you get when you add all the numbers together and divide by the number of numbers in the list. This is typically what people mean when they simply say "average." Notation for mean: \bar{x} .
- Box & Whisker Plot--->A type of graph that helps view a list of numbers auickly to see their spread.

Given a list of numbers, this is how to create a box and whisker plot:

- (1) Put the numbers in decreasing order.
- (2) Find the following information.
 - (a) The lowest number in the list.
 - (b) The lower quartile, which is the median of the lowest half of the numbers.
 - (c) The median of the list of numbers.
 - (d) The upper quartile, which is the median of the largest half of the numbers.
 - (e) The highest number in the list.
- (3) Plot these 5 numbers on a number line.
- (4) Make a box from the lower quartile to the upper quartile, and indicate the median in that box with a vertical line segment where the median occurs.
- (5) Connect the lowest number to the lower quartile with a line segment and connect the upper quartile to the highest number with a line segment.

The box represents where 50% of the list of numbers lie. The whiskers 9b represent about 25% of the other numbers.

- IQR (Interquartile Range)—>The difference between the upper and lower quartile. For example, if the upper quartile is 93 and the lower quartile is 76, then the IQR is 93 76 = 17.
- Outlier-->Any value of data that lies more than 1.5 IQR units below the lower quartile OR more than 1.5 IQR units above the upper quartile.
 - n^{th} <u>Percentile</u>-->A number is in the n^{th} percentile of some data if it is greater than or equal to n% of the data.

Consider the data set 1, 1, 3, 3, 3, 4, 4, and 5. The mean, or average value, for this set is the sum of the values. 24, divided by 8, or 3. We can represent these data by using stacks of unit blocks as weights on a number line to illustrate the idea of

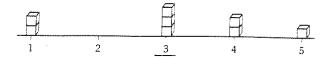


FIGURE 8.17

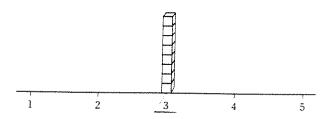


FIGURE 8.18

the arithmetic mean as a clustering or balance point. On the number line shown in Figure 8.17, a unit block on 5, for example, represents the number 5 from the data set. Two unit blocks on 4 represent the two 4's from the data set, and so on. The fulcrum is at 3, the mean of the data set.

To demonstrate visually the clustering at a mean, we move blocks to the center while maintaining the balance by compensating moves of blocks on the left side to the right and moves of blocks on the right side to the left. Specifically, we move the two blocks at 1 each 2 units to the right, a total of 4 units to the right. To maintain the balance, we move the two blocks at 4 each a unit to the left, effecting a movement of 2 units to the left. We also move the block at 5, 2 units to the left, making a total of 4 units moved to the left. As we make the same number of moves of unit blocks to the right as to the left, the number line remains in balance about its mean of 3, as shown in Figure 8.18. Thus, the mean can be considered the "balance point" or "cluster point" for the numbers in the data set it represents.

Complete the estimation application by using the idea of the mean as a balance or cluster point.

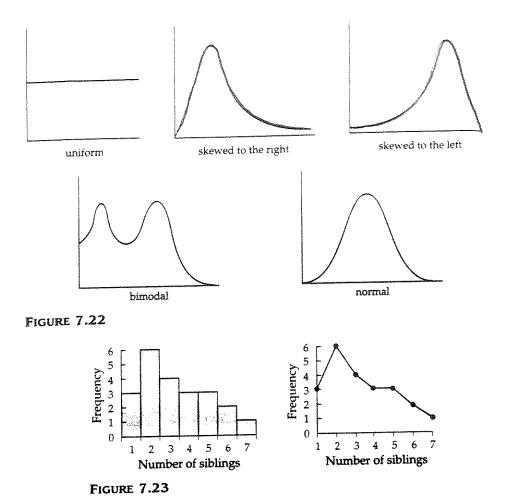


Table 7.12 gives one example for each of the five distributions.

TABLE 7.12

x axis (independent variable)	y axis (dependent variable
Months of the year	Temperature in Hawaii
Salaries in a factory	Frequency
Class sizes in a high school	Frequency
Time to finish an exam	Frequency
Hours that Battery A lasted	Frequency
	Months of the year Salaries in a factory Class sizes in a high school Time to finish an exam

A data distribution can be classified as approximately symmetric (for example, the normal distribution) or skewed in one direction. When a nonsymmetric curve has a longer left-hand "tail," as shown in Figure 12-22(a), it is skewed to the left. A curve with a longer "tail" on the right, as shown in Figure 12-22(b), is skewed to the right.

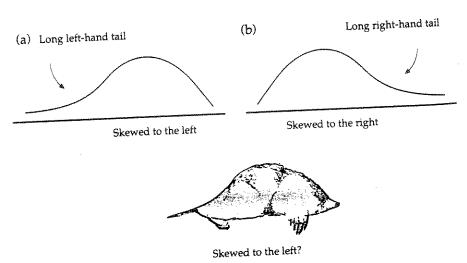


Figure 12-22

LE 14 Connection

Decide whether each set of discrete data is most likely to be approximately symmetric. skewed to the left, or skewed to the right.

- (a) Second grade students' test scores on a fourth grade test
- (b) Fourth grade students' test scores on the same test
- (c) Sixth grade students' test scores on the same test

Math4020 10.2-2 Notes Measuring Dispersion

Since the range of data doesn't give enough information about the distribution of the data, we need to look at other attributes of the data.

<u>Variance</u>-->The mean of the squared differences between each number and the mean of the list of numbers. For a list of numbers,

and the filedit of the har of the variance =
$$\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n}$$

Standard Deviation—>The square root of the variance, usually denoted σ , i.e. $\sigma = \sqrt{variance}$.

<u>Z-score</u>-->For a particular number x, compared to a list of data values, the z-score is calculated with the following formula.

$$z = \frac{x - \overline{x}}{\sigma}$$
 Where σ = standard deviation of the list of data,

and \bar{x} = the mean of that data set.

The z-score indicates how many standard deviations x is away from the mean.

$$z > 0 ==> x$$
 is above the mean, i.e. $x > \overline{x}$.

$$z < 0 ==> x$$
 is below the mean, i.e. $x < \overline{x}$.

$$z = 0 = x$$
 is exactly the mean, i.e. $x = \overline{x}$.

<u>Distribution</u>—>Graph of the data values (on horizontal axis) vs. relative frequency of each number in the data (on vertical axis), on a Cartesian coordinate system.

Relative Frequency—>If we have n numbers in our data list, then the relative frequency of any x in the list is (the number of times x occurs in the list)/ n.

Bell-shaped Distribution-->A smooth, continuous distribution whose shape resembles a bell shape. (It's like a continuous histogram, or a "histogram-gone-smooth.")

(1) The median will occur at the x-value where the vertical line cuts the region under the curve into two equal-area regions.

(2) The mode will occur at the x-value where the highest point on the curve exists.

(3) The mean will occur at the x-value where the distribution would be balanced (like a teeter-totter).

Normal Distribution-->A special bell-shaped distribution that's symmetric and the mean=mode=median. Shape is determined by the mean and standard deviation (the larger the standard deviation, the flatter the normal curve will be).

For a normal distribution,

~68% of the data is in the interval $[\bar{x}-\sigma,\bar{x}+\sigma]$

~95% of the data is in the interval $[\bar{x}-2\sigma,\bar{x}+2\sigma]$

~99.7% of the data is in the interval $[\bar{x}-3\sigma, \bar{x}+3\sigma]$

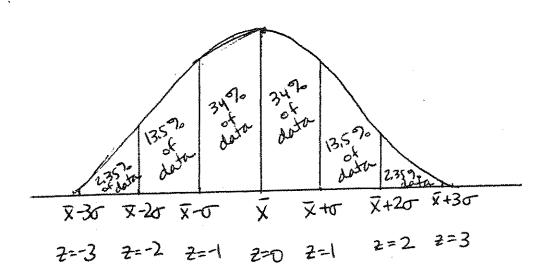
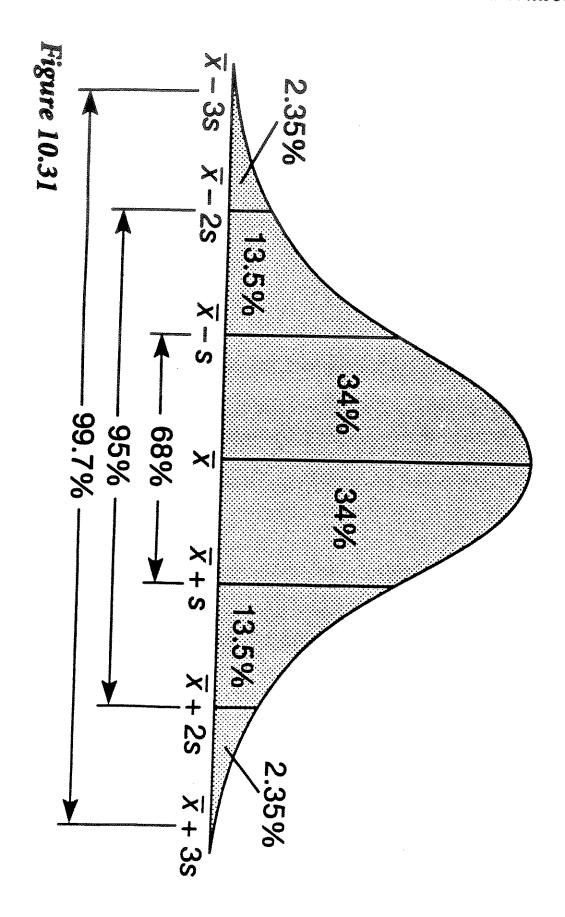


TABLE 10.15

PERCENTILE	z-SCORE	PERCENTILE	z-SCORE	PERCENTILE	z-SCORI
1	-2.326	34	-0.412	67	0.44
2	-2.054	35	-0.385	68	0,468
3	-1.881	36	-0.358	69	0.496
4	-1.751	37	-0.332	70	0.524
5	-1.645	38	-0.305	71	0.553
6	-1.555	39	-0.279	72	0.583
7	-1.476	40	-0.253	73	0.613
8	-1.405	41	-0.228	74	0.643
9	-1.341	42	-0.202	75	0.674
10	-1.282	43	-0.176	76	0.706
11	-1.227	44	-0.151	77	0.739
12	-1.175	45	-0.126	78	0.772
13	-1.126	46	-0.1	79	0.806
14	-1.08	47	-0.075	80	0.842
15	-1.036	48	-0.05	81	0.878
16	-0.994	49	-0.025	82	0.915
17	-0.954	50	0	83	0.954
18	-0.915	51	0.025	84	0.994
19	-0.878	52	0.05	85	1.036
20	-0.842	53	0.075	86	1.08
21	-0.806	54	0.1	87	1.126
22	-0.772	55	0.126	88	1.175
23	-0.739	56	0.151	89	1.227
24	-0.706	57	0.176	90	1.282
25	-0.674	58	0.202	91	1.341
26	-0.643	59	0.228	92	1.405
27	-0.613	60	0.253	93	1.476
28	-0.583	61	0.279	94	1.555
29	-0.553	62	0.305	95	1.645
30	0.524	63	0.332	96	1.751
31	-0.496	64	0.358	97	1.881
32	-0.468	65	0.385	98	2.054
33	-0.44	6 6	0.412	99	2.326



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Math4020 11.1 Notes Probability

Experiment --> The act of making an observation.

Outcome -- > One of the possible things that can occur.

<u>Sample Space</u>-->The set of all possible outcomes (usually denoted as \$).

Event->A subset of the sample space (usually denoted as E).

Probability-->The relative frequency we expect an event to occur (can be written as a fraction, decimal, ratio or percent)

 $P(E) = \frac{n(E)}{n(S)}$ That is, the probability of an event E is the number of ways the event E can happen divided by the number of outcomes of S.

Since $\emptyset \subseteq E \subseteq S$, and we know that the number of elements in the empty set is 0, i.e. $n(\emptyset) = 0$, then

 $0 \le n(E) \le n(S)$. And dividing by n(S), we get

$$\frac{0}{n(S)} \le \frac{n(E)}{n(S)} \le \frac{n(S)}{n(S)}$$
 . Finally, we have

 $0 \le P(E) \le 1$. In other words, the probability of any event is always a number between 0 and 1 (inclusive).

<u>Experimental Probability</u>-->The probability calculated from an actual experiment. The probability for the same event may vary from observation to observation.

<u>Theoretical Probability-->This is based on ideal occurrences (usually what we mean when we say "probability").</u>

The probability of A **OR** B is equal to the probability of A plus the probability of B minus the probability of A **AND** B (that got counted twice).

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

The probability of the the event not happening equals 1 minus the probability of the event happening.

$$P(\bar{E})=1-P(E)$$

Math4020 11.2 Notes Probability

Fundamental Counting Property-->If an event A can occur in r ways and for all of the r ways, an event B can occur in s ways, then the event A & B can occur, in succession, in rs ways. (This can help when calculating probability.)

Example 1: At a friend's birthday party, she had three different flavors of ice cream (vanilla, chocolate, or mint), two toppings (chocolate or strawberry sprinkles), and we could have our ice cream in a cone or a cup. I wanted to have one scoop of ice cream with a topping.

(a) How many choices did I have?

(b) If I randomly made a choice, what is the probability that I got mint ice cream with chocolate sprinkles on a cone?

Example 2: What is the probability of getting a sum of 8 on a pair of dice?

<u>Tree Diagram</u>-->This can be used to represent outcomes of an experiment (rather than writing out each element in a set).

Example 3: A toy company makes tiles for a game. Each tile has these attributes-- (1) it's either blue or green

(2)it's a square, triangle or circle shape

(3) it is dotted or striped.

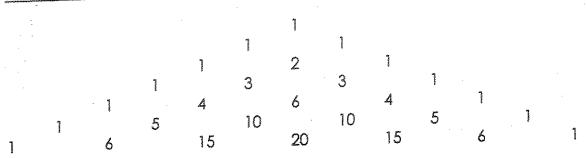
We can use a tree diagram to draw all the possible different tiles that can be made with these attributes.

<u>Probability Tree Diagram</u>-->A tree diagram that has the probabilities of each event listed on the branches.

Example 4: I have a drawer with loose socks. In total, I have 5 blue socks, 3 black socks and 2 red socks. I pull one sock from the drawer, at random, and then another sock, trying to find a match.

Draw a probability tree diagram that represents this experiment.

<u>Pascal's Triangle--></u>



Example 5: Draw a tree diagram to represent tossing a coin 5 times (and recording whether it lands on Heads or Tails).
What does this have to do with Pascal's Triangle?

Additive Property of Probability Tree Diagrams-->For pairwise mutually exclusive events (events that have no dependence on one another) $E_1, E_2, E_3, \dots, E_n$ (all from the same sample space S), if $E = E_1 \cup E_2 \cup E_3 \cup \dots \cup E_n$, then $P(E) = P(E_1) + P(E_2) + P(E_3) + \dots + P(E_n)$.

That is, we can add probabilities at the end of the probability tree diagram to get the probability of the event we want.

Multiplicative Property of Probability Tree Diagrams—>If an experiment consists of a sequence of simpler experiments and can be represented by a probability tree diagram, then the probability of any of the simpler experiments is the product of all probabilities on its branch.

Example 6: We have five candies in a jar—2 white, 2 red, and 1 blue. We pull out one candy, record its color, replace it, then draw another candy and record its color.

(a) Draw a probability tree diagram for this experiment.

(b) What is the probability that both candies are the same color?

(c) What is the probability that the first candy is white and the second candy is red?

Example 7: Refer to Example 4--

(a) What is the probability that I choose a blue pair of socks?

(b) What is the probability that I choose a matching pair of socks?

(c) What is the probability that I choose a red and a blue sock?

Example 8: Refer to Example 6—We have the same candles in a jar, but this time, we will not replace the first candy we draw. End the experiment as soon as a red candy is drawn.

(a) Draw a probability tree diagram for this experiment.

(b) Given event A = only 1 draw is needed to get a red candy event B = two draws are needed to get a red candy event C = three draws are needed to get a red candy

(i) Find P(A).

(ii) Find P(B).

(iii) Find P(C).

(iv) Find $P(A \cup B)$. In words, what is this asking for?

Math4020 11.4 Notes Permutations and Combinations

<u>Permutation</u>—>An ordered arrangement of objects.

Combination->A collection of objects, in no particular order.

Factorial $\rightarrow n! = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 3 \cdot 2 \cdot 1$ By definition, 1! = 1 and 0! = 1.

The number of permutations of n distinct objects, taken all together, is nl, i.e. "n factorial."

Example 1: Simplify the following expressions.

- (a) $\frac{10!}{3!}$
- (b) $\frac{3!5!}{2!}$

Let's make a table to record these next several results and try to generalize them to a formula.

Permutation or Combination	# things to choose from	# things we chose	# permutations or combinations	Formula guess
			12.	
			72	

Example 2: How many 4-letter "words" can we form (where we count any distinct grouping of letters as a word) with no repeating letters.	
Permutation or Combination?	
Example 3: I want to arrange three marbles in a row. In how many ways can this be done?	
Permutation or Combination?	
Example 4: If 6 horses are entered in a race and there can be no ties, how many different orders of finish are there?	
Permutation or Combination?	

Example 5: In your sorority, you need to choose a president, treasurer and secretary out of a group of 15 women. How many ways can this occur? Permutation or Combination? Example 6: My friend made 5 cakes and she has offered to let me take three of them home. How many different groups of cakes can I choose? Permutation or Combination? Example 7: I have 10 marbles and I want to choose 4 to give to my best friend. How many different groupings of marbles can I give her? Permutation or Combination?

Number of Permutations:

The number of permutations of r objects chosen from n objects,

where $0 \le r \le n$, is

$$_{n}P_{r}=\frac{n!}{(n-r)!}$$
 read "n permute r."

Number of Combinations:

The number of combinations of r objects chosen from n objects,

where $0 \le r \le n$, is

$$_{n}C_{r} = \frac{n!}{(n-r)!r!} = \left(\frac{n}{r}\right)$$
 read "n choose r."

 $= \frac{number\ of\ permutations}{number\ of\ permutations\ per\ combination}$

 $= \frac{number\ of\ permutations}{(number\ chosen)!}$

Example 9: You have a class of 20 students. You need to select two students to be hall monitors. How many groups of two students can you choose from your class?

Permutation or Combination?

Example 10: I go to an ice cream store that has 30 flavors of ice cream. (a) I want to get a bowl with three scoops to eat. How many different groupings of three flavors can I get? Permutation or Combination? (b) I want to get my three scoops on a cone instead. How many different ways can I arrange three flavors on my cone? Permutation or Combination? Example 11: I want to choose 3 candies out of 5 different candies. How many choices do I have? Permutation or Combination?

Example 12: I have 4 different cookies. How many ways can I put them in a line?
Permutation or Combination?
Example 13: How many 12-person juries can be chosen from 30 candidates?
Permutation or Combination?
Example 14: On a 10-question True/False test, how many ways can 8 or more answers be correct?
Cau a of those grawers be conecia

Example 15: At a pizza restaurant, you have 10 choices for toppings, 3 sauce choices, and 4 types of crust. They are running a special today for \$6 you can get a 1-topping pizza with your choice of topping, sauce and crust. How many different pizzas can be ordered?

Permutation or Combination?	
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Example 16: For a family of 9 children, with 4 girls, how many boygirl arrangements can there be?

Permutation or Combination?	
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Example 17: You toss a coin 6 times and record the result—either H=heads or T=tails.				
Permutation or Combination?				
(a) How many ways can you get 2 heads?				
(b) How many ways can you get 5 tails?				
Example 18: You have 3 red (triangular) flags, 2 green flag blue flags. How many different ways can you order the floyour flagpole?	s and 4 ags on			
Permutation or Combination?				

Math4020 11.3 Notes Odds & Conditional Probability

Odds-->The odds in favor of an event E are given by $n(E):n(\bar{E})$. That is, the odds in favor of event E is the ratio of the number of ways E can occur to the number of ways E does not occur. The odds against event E are given by $n(\bar{E}):n(E)$.

Example 1: When drawing a card from a deck of cards, what are the odds in favor of

- (a) getting a black card.
- (b) getting a face card.
- (c) getting a 2.
- (d) getting the 5 of hearts.

Example 2: In the game of craps, one wins on the first roll of the pair of dice if a 7 or 11 is thrown (i.e. the sum of the dice is 7 or 11). What are the odds of winning on the first roll?

Given the probability of event E, P(E) , determine the odds in favor of E and the odds against E.

We know the odds in favor of E are given by

$$\frac{n(E)}{n(\overline{E})} = \frac{n(E)}{n(\overline{E})} \cdot \frac{1/n(S)}{1/n(S)}$$

$$= \frac{P(E)}{P(\overline{E})}$$

$$= \frac{P(E)}{1 - P(E)}$$

$$= \frac{P(E)}{1 - P(E)}$$

or P(E):1-P(E)

Likewise, the odds against E are given by 1-P(E): P(E)

Thus, if we know the probability of E (or the probability of its complement), then we can get to the odds in favor or against E. This is useful, especially for unequally likely outcomes.

Example 3: Find the odds in favor of an event E if

(a)
$$P(E) = \frac{1}{2}$$

(b)
$$P(E) = \frac{2}{5}$$

(c)
$$P(E) = \frac{3}{7}$$

Example 4: If $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, and $P(C) = \frac{1}{6}$ and events A, B and C are mutually exclusive, compute the odds in favor of A or C.

Example 5: Three coins are tossed.

- (a) What are the odds in favor of getting two heads and one tail?
- (b) What are the odds against getting three heads?

Example 6: A jar contains 5 yellow, 4 blue, and 8 green marbles. What are the odds in favor of selecting a yellow marble if a single marble is drawn from the jar?

Example 7: The odds against the Heatwaves winning the Crabgrass Derby are 7 to 1. Express the probability of winning.

Conditional Probability-->Suppose A and B are events in sample space S such that $P(B)\neq 0$. The conditional probability that event A occurs, given that B occurs, is

$$+ P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Example 8: A family has three children. Let A be the event that they have a girl first and B be the event that they have a boy second.

- (a) Draw a Venn Diagram to represent A and B.
- (b) Use the Venn Diagram to calculate the following:
 - (1) P(A|B)
 - (2) P(B|A)

Example 9: All 24 students in Ms. Henry's preschool class are either three or four years old, as shown in the following table. A student is

selected at random.

	30100100			
- proprogramma		Age Three	Age Four	
-	Boys	8	3	
	Girls	6	7	

(a) What is the probability that the student is three years old?

(b) What is the probability that the student is three years old, given that a boy was selected?

(c) What is the probability that the student is three years old, given that a girl was selected?

Example 10: Five black balls numbered 1, 2, 3, 4, and 5 and seven white balls numbered 1 through 7 are placed in a bag. One is chosen at random.

(a) What is the probability it is numbered 1 or 2?

(b) What is the probability it is numbered 5 or is white?

- (c) What is the probability it is numbered 5, given that it is white?
- (d) What is the probability it is numbered 3, given that it is black?
- (e) What is the probability it is black, given that it's numbered 2?
- (f) What are the odds in favor of getting a black ball?

Example 11: Mrs. Ricco has seven brown-eyed and two blue-eyed brunettes in her fifth grade class. She also has eight brown-eyed and three blue-eyed children with black hair. A child is selected at random.

(a) What is the probability that the child is a brown-eyed brunette?

(b) What is the probability that the child has brown eyes or is brunette?

(c) What is the probability that the child has brown eyes, given that he or she is a brunette?

(d) What is the probability that the child has black hair, given that the child has blue eyes?

(e) What are the odds in favor of the child being a brunette?

(f) What are the odds against the child having brown eyes?

Example 12: Do book problem 11.3 A #20.

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Math4020 Final Exam Topics

From Chapters 10 and 11

- Statistics Graphs
 - Histograms
 - Stem and Leaf Plots
 - O Bar Graphs
 - Pie Charts (a.k.a. Circle Graphs)
 - O Scatter Plots
 - o Line Graphs
 - o Line Plots
- Central Tendencies
 - o Mean
 - Median
 - o Mode
 - Standard Deviation
 - o Z-scores/percentile
- Probability
 - Theoretical and Experimental
 - O Tree Diagrams
 - Fundamental Counting Property
 - o Odds
 - Conditional Probability
 - o Permutations
 - Combinations

From Chapters 12 and 13

- Classifying/recognizing 2d polygons/shapes
- Symmetry
- Corresponding angles
- Alternate interior angles
- Regular Polygons
 - Vertex angles (a.k.a. interior angles)
 - o Central angles
 - Exterior angles
- Angle sum for a polygon
- Tessellations
- Converting units of measurement
- Perimeter of 2d shapes

- Area of 2d shapes
- Pi/circles
- Pythagorean theorem
- Classifying/recognizing 3d polyhedron/solids
- Platonic solids
- Surface area of 3d solids
- Volume of 3d solids
- Scaling

From Chapters 14 and 15

- Basic Euclidean Constructions
- Congruence of Triangles (SSS, SAS, ASA)
- Similarity of Triangles (AA, SSS, SAS)
- Algebra/Geometry Tie-in
 - o Slope of a line (parallel and perpendicular lines slopes)
 - o Equation of a line
 - o Midpoint
 - Distance between two points
 - Collinearity
 - Equation of a Circle

Bring to Final Exam:

- o z-score sheet
- o conversion sheet
- o 4x ise to with your notes
- o straightedge
- o compass
- o calculator
- o pencils and/or pens