Chapter 4 Review

4.1

For each function, find its inverse.

1. $f(x) = 2(x + 3)^3 - 4$
2. $h(x) = \dfrac{\sqrt[5]{2x - 1}}{3}$
3. $f(w) = \dfrac{3w}{4w + 9}$
4. $p(y) = 6y^{\frac{-1}{3}} + 2$
5. $g(x) = \dfrac{x^3 + 1}{5 - x^3}$
6. $y(x) = \sqrt[3]{\dfrac{x + 7}{2x - 11}}$
7. $f(p) = \dfrac{-2}{p + 1}$
8. For each function, restrict the domain so it has an inverse.Then, find the inverse function.
   a. $f(x) = 5(x - 1)^2 + 3$
   b. $y(x) = \sqrt{\dfrac{x}{2}} - 1$
   c. $h(x) = -2x^4 + 7$

4.2

Sketch the graph of each function. Label the y-intercept.

9. $y = e^x + 2$
10. $y = 5^x - 1$
11. $f(x) = 3^{-x}$
12. $g(x) = -2^x + 3$
13. $y = 4^{\frac{x}{2}}$

Simplify each expression.

14. $(3^{4-x})^{3x}$
15. $\dfrac{4^{x+1}}{4^{3-x}}$
16. $\dfrac{\pi^{5x+3}}{\pi^2}$
17. $(-6^2 e)^x$
18. $\dfrac{\pi^{2x} e^{3y}}{e^{2x} \pi^{3y}}$

4.3

Rewrite the logarithmic statement as its equivalent exponential statement using the definition of logarithm.

19. $\log_5 25 = 2$
20. $\log_4 \left(\dfrac{1}{4}\right) = -1$

21. $\log 1 = 0$

22. $\ln\left(\frac{1}{\sqrt{e}}\right) = -\frac{1}{2}$

Simplify each expression, without using a calculator.

23. $\log_8 1$

24. $\log_2 16$

25. $\ln e^{-5}$

26. $\log_3\left(\frac{1}{81}\right)$

For each function, find the domain and sketch the graph. Label the x-intercept.

27. $f(x) = \log_3(-x)$

28. $y = \log(x - 2)$

29. $g(x) = \ln(x) + 3$

30. $h(x) = 2\log_5(x) - 1$

31. $y = -\log_2(x + 1)$

4.4

Use properties of logarithms to expand each expression completely.

32. $\log\left(\frac{x^2 + 5}{x^3}\right)^4$

33. $\ln\left(x^2\sqrt[3]{\frac{(x-1)^4}{x+9}}\right)$

34. $\log_5\left(\frac{(x+1)^2(x-2)^3(x+3)^4}{x}\right)$

Use properties of logarithms to condense each expression completely.

35. $3\log_2 x - 4\log_2(x + 5) + \log_2 9$

36. $\frac{1}{2}\log_4(x - 3) + \frac{1}{4}\log_4 x - \log_4(\sqrt{x})$

37. $3\ln(e^x) - \ln x^2 + \ln(5x - 2)$

38. $\log(4x) + \log(5y) - \log(xy)$

39. $\log_5(x^2 - 5) + 3\log_5 x - \log_5\left(\frac{x}{2}\right)$

Evaluate each expression, without using a calculator.

40. $3\ln e - \ln 1 + \ln\left(\frac{1}{\sqrt{e}}\right) + \ln e^5$

41. $\log_4\left(\frac{120(24)}{18(40)}\right) - \log_5 625 + \log 1000$

42. $\log_2(2^\pi) + \log_\pi(\pi^2) - \ln(e^3)$

4.5

Solve each equation.

43. $2e^{5x} - 3 = 9$

44. $5^{x^2}5^{3x} = 5^{10}$

45. $3^{x+1} + 2 = 4(3^{x+1}) - 7$

46. $2^{3w+5} + 1 = 2^0 + \frac{1}{16}$

47. $2(e^{2x} - 10) = -3e^x$

48. $\log_4 5 + \log_4 x = \log_4(3x + 10)$

49. $\log_2 x^2 - \log_2(x + 5) = 2$

50. $\log x + \log(x + 5) = \log 66$

51. $e^{\ln(x^2+x)} = 90$

52. $\ln(x + 2) = \ln(x + 1) + 3$

53. $3 = \log_5(x + 2) + \log_5\left(\frac{1}{x}\right)$

54. $2x10^x = x^2 10^x$

4.6

55. Suppose the number of rabbits on a small island is given by $N = 400(0.1^{0.2^t})$, where t is the number of years after 2005.

a. How many rabbits are on the island in 2005?

b. How many rabbits are on the island in 2006?

c. In the year 2020, what will the rabbit population be?

56. The number N of people in a community who are reached by a particular rumor at time t (in days) is given by $N(t) = \dfrac{680}{1 + 169e^{-0.4t}}$.

a. Find $N(0)$, the number of people who initially know the rumor.

b. How long will it take 340 people to know the rumor?

57. The demand function for a product is given by $p = 180(4^{\frac{-q}{15}})$.

a. At what price will there be 2 units demanded?

b. If the unit price is \$110, how many units will be demanded?

58. The half-life of radioactive radium is 1620 years. What percent of a present amount of radioactive radium will remain after 870 years?

59. The number of units of a product sold after t years is given by $n(t) = 150(0.2^{0.03t})$. After how many years will there be 125 units sold?

60. The quantity, measured in milligrams, of a radioactive substance present after t years is given by $q = 180e^{-0.04t}$. After how many years will there be 14 mg present?

61. The population of Mathville, with initial population of 14,000 in the year 2000, grows at a rate of 3% per year.

a. What is the population function?(Let P = population and t = number of years past year 2000.)

b. What is the population in 2002?

c. In what year will the population be 35,000?

62. Melida saved \$5,000 from her weekly cash over the last two years. She wants to invest her money in an account that grows according to the formula $A(t) = P(1.05^{2t})$, where P is the original amount invested, t is the number of years she leaves her money in that account and A is the value of that account at time t.

a. How much will the account be worth after two years?

b. After how many years will she have \$8,500?

CHAPTER 4 REVIEW ANSWER KEY

1. $f^{-1}(x) = \sqrt[3]{\frac{x+4}{2}} - 3$

2. $h^{-1}(x) = \frac{(3x)^5 + 1}{2}$

3. $f^{-1}(w) = \frac{9w^2}{3 - 4w}$

4. $p^{-1}(y) = \frac{216}{(y-2)^3}$

5. $g^{-1}(x) = \sqrt[3]{\frac{5x-1}{x+1}}$

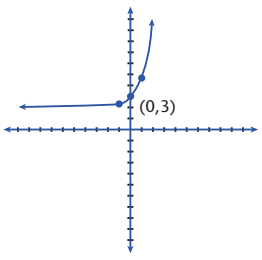
6. $y^{-1}(x) = \frac{7 + 11x^3}{2x^3 - 1}$

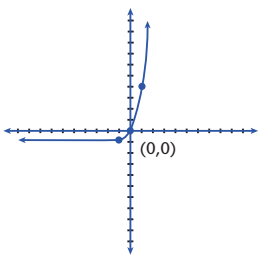
7. $f^{-1}(p) = -1 - \frac{2}{p}$

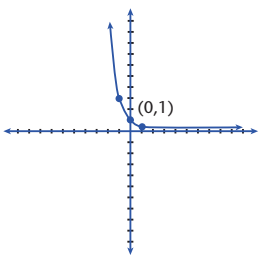
8. **a.** domain: $x \geq 1$; $f^{-1}(x) = \sqrt{\frac{x-3}{5}} + 1$

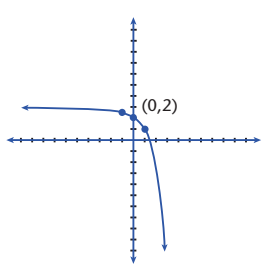
   b. domain: $x \geq -1$; $y^{-1}(x) = 2(x+1)^2$

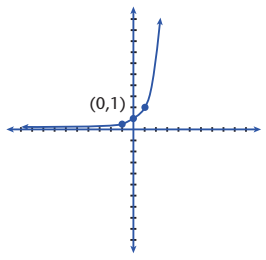
   c. domain: $x \geq 0$; $h^{-1}(x) = \sqrt[4]{\frac{7-x}{2}}$

9. 

10. 

11. 

12. 

13. 

14. $3^{12x - 3x^2}$

15. 4^{2x-2}

16. π^{5x+1}

17. $(-36)^x e^x$

18. $\left(\frac{\pi}{e}\right)^{2x-3y}$

19. $5^2 = 25$

20. $4^{-1} = \frac{1}{4}$

21. $10^0 = 1$

22. $e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$

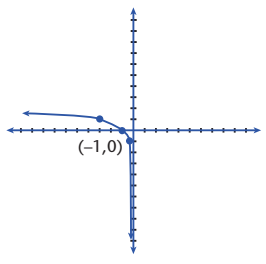
23. 0

24. 4

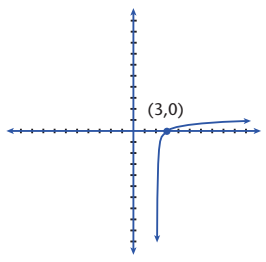
25. –5

26. –4

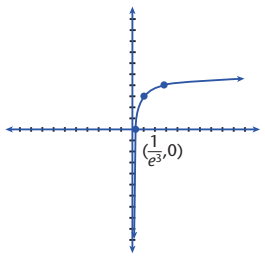
27. domain: $x < 0$



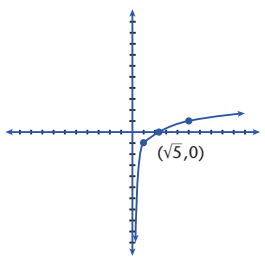
28. domain: $x > 2$



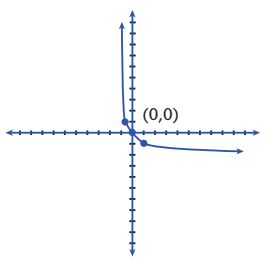
29. domain: $x > 0$



30. domain: $x > 0$



31. domain: $x > -1$



32. $4\log(x^2 + 5) - 12\log x$

33. $2\ln x + \frac{4}{3}\ln(x-1) - \frac{1}{3}\ln(x+9)$

34. $2\log_5(x+1) + 3\log_5(x-2) + 4\log_5(x+3) - \log_5 x$

35. $\log_2\left(\frac{9x^3}{(x+5)^4}\right)$

36. $\log_4\left(\frac{\sqrt{x-3}}{\sqrt[4]{x}}\right)$

37. $\ln\left(\frac{e^{3x}(5x-2)}{x^2}\right)$

38. $\log(20)$

39. $\log_5(2x^2(x^2-5))$

40. 7.5

41. 0

42. $\pi - 1$

43. $x = \frac{\ln 6}{5}$

44. $x = 2, -5$

45. $x = 0$

46. $w = -3$

47. $x = \ln\left(\frac{5}{2}\right)$

48. $x = 5$

49. $x = 2 \pm 2\sqrt{6}$

50. $x = 6$

51. $x = 9, -10$

52. $x = \frac{e^3 - 2}{1 - e^3} \approx -0.9476$

53. $x = \frac{1}{62}$

54. $x = 0, 2$

55. **a.** 40
b. ~252
c. 400

56. **a.** 4
b. ~13 days

57. **a.** \$149.62
b. ~5 units

58. 68.9%

59. ~4 years

60. ~64 years

61. **a.** $P = 14000e^{0.03t}$
b. ~14866
c. 2031

62. **a.** \$6077.53
b. ~5.44 years