1. Write the equation of the line that passes through the point (5, -3) and is perpendicular to the line $2x - 3y = 6$. (Put answer in slope-intercept form.)

$$-3y = -2x + 6$$
$$y = \frac{2}{3}x - 2$$

$\Rightarrow$ we want slope $m = \frac{-3}{2}$

thru $(5, -3)$
$\quad x_1 \quad y_1$

$$y - y_1 = m(x - x_1)$$
$$y - (-3) = \frac{-3}{2}(x - 5)$$
$$y + 3 = \frac{-3}{2}x + \frac{15}{2}$$
$$y = \frac{-3}{2}x + \frac{15}{2} - \frac{6}{2}$$
$$y = \frac{-3}{2}x + \frac{9}{2}$$

Line: $y = \frac{-3}{2}x + \frac{9}{2}$

2. Margie makes a $500 contribution at the end of each quarter to a retirement account for 10 years earning 7% interest.

(a) How much money is in the account after the 10 years of contributions?

FV ordinary annuity

$t = 10$ $n = 4$ $N = 40$

$r = 0.07$ $r_c = \frac{0.07}{4} = 0.0175$

$S = R\left(\frac{(1+r_c)^N - 1}{r_c}\right)$

$R = 500$

$S = 500\left(\frac{1.0175^{40} - 1}{0.0175}\right) = \$28617.07$

Answer: $28,617.07

(b) After the first 10 years, she makes no additional contributions and no withdrawals, and she leaves the money in the account for another 10 years. How much money is in the account at the end of those next 10 years?

Compound interest

$S = P(1+r_c)^N$

$N = nt = 10(4) = 40$

$r_c = 0.0175$

$P = 28617.07$

$S = 28617.07(1.0175^{40})$ $57,279.85

Answer: $57,279.85

3. Mr. Rogers makes and sells sweaters. His fixed costs are \$1200 and it costs \$8 to make each sweater. His revenue function is described by $R(x) = -x^2 + 88x$.

(a) How many sweaters should Mr. Rogers make to maximize profit?

x = # sweaters

$C(x) = 1200 + 8x$

$P(x) = R(x) - C(x)$

$= -x^2 + 88x - (1200 + 8x)$

concave down parabola $\rightarrow P = -x^2 + 80x - 1200$

max at vertex at $x = \frac{-80}{2(-1)} = 40$

Number of sweaters: 40

(b) What is Mr. Rogers' maximum profit?

$P(40) = -(40^2) + 80(40) - 1200$

$= -1600 + 3200 - 1200$

$= 1600 - 1200$

$= 400$

Maximum profit: \$400

(c) How many sweaters must he sell to break even?

break even when $P = 0$

$-x^2 + 80x - 1200 = 0$

$x^2 - 80x + 1200 = 0$

$(x - 20)(x - 60) = 0$

$x = 20, 60$

Number of sweaters to break even: 20 or 60

4. Solve the following system of equations using Gaussian elimination.

$$\begin{aligned} 2x+4y \quad &= -4 \\ 6x \quad +3z &= 12 \\ y+z &= -2 \end{aligned}$$

$$\begin{array}{c} (\frac{1}{2}) \\ (\frac{1}{3}) \\ \\ \end{array} \left[\begin{array}{ccc:c} 2 & 4 & 0 & -4 \\ 6 & 0 & 3 & 12 \\ 0 & 1 & 1 & -2 \end{array}\right] \qquad \begin{array}{c} (-2) \\ \\ \\ \end{array} \left[\begin{array}{ccc:c} 1 & 2 & 0 & -2 \\ 2 & 0 & 1 & 4 \\ 0 & 1 & 1 & -2 \end{array}\right]$$

$$\begin{array}{c} \\ \\ (4) \end{array} \left[\begin{array}{ccc:c} 1 & 2 & 0 & -2 \\ 0 & -4 & 1 & 8 \\ 0 & 1 & 1 & -2 \end{array}\right] \qquad \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \left[\begin{array}{ccc:c} 1 & 2 & 0 & -2 \\ 0 & 0 & 5 & 0 \\ 0 & 1 & 1 & -2 \end{array}\right]$$

2 $5z=0$

$z=0$

3 $y+z=-2$

$y+0=-2$

$y=-2$

1 $x+2y=-2$

$x-4=-2$

$x=2$

Solution: $(2, -2, 0)$

5. Solve these equations.

(a) $\log_3(x+4)-\log_3(x+1)=\log_3 x$

$$\log_3\left(\frac{x+4}{x+1}\right)=\log_3 x$$

$$\frac{x+4}{x+1}=x$$

$$x+4=x(x+1)$$

$$x+4=x^2+x$$

$$-x \qquad -x$$

$$4=x^2$$

$x=\pm 2$

but if $x=-2$

$\log_3(-2+1)$ DNE

$\Rightarrow x=2$ only soln

x = 2

(b) $150e^{0.05t}-40=410$

$$150e^{0.05t}=450$$

$$e^{0.05t}=3$$

$$\ln 3=0.05t$$

$$t=\frac{\ln 3}{0.05}$$

t = $\frac{\ln 3}{0.05}\simeq 21.97$

(c) $\frac{2x+1}{x-2}+4=\frac{5}{x-2}$

$$\frac{\cancel{(x-2)}(2x+1)}{\cancel{(x-2)}}+4(x-2)=\frac{5\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$2x+1+4x-8=5$$

$$6x-7=5$$

$$6x=12$$

$$x=2$$

but $x\neq 2$
(because it makes the denominator zero)

x = N.S.

6. You are in the business of manufacturing nordic skis. You make two kinds of skis, classic and skate. It costs you \$100 to make a pair of classic skis, and \$150 to make a pair of skate skis. Your daily production budget is \$15,000. You need to make at least as many classic skis as skate skis, and you can't put out more than 100 pairs of classic skis in a day.

(a) Write down the inequalities that describe your constraints in making skis. Let x denote the number of pairs of classic skis and y the number of pairs of skate skis.

$x \geq 0,\ y \geq 0$	
$100x + 150y \leq 15000$	① $y \leq -\frac{2}{3}x + 100$
$x \geq y$	② $y \leq x$
$x \leq 100$	③ $x \leq 100$

(b) Graph these inequalities, label the axes, shade in the feasibility region, and label the vertices.

∴ ① & ②

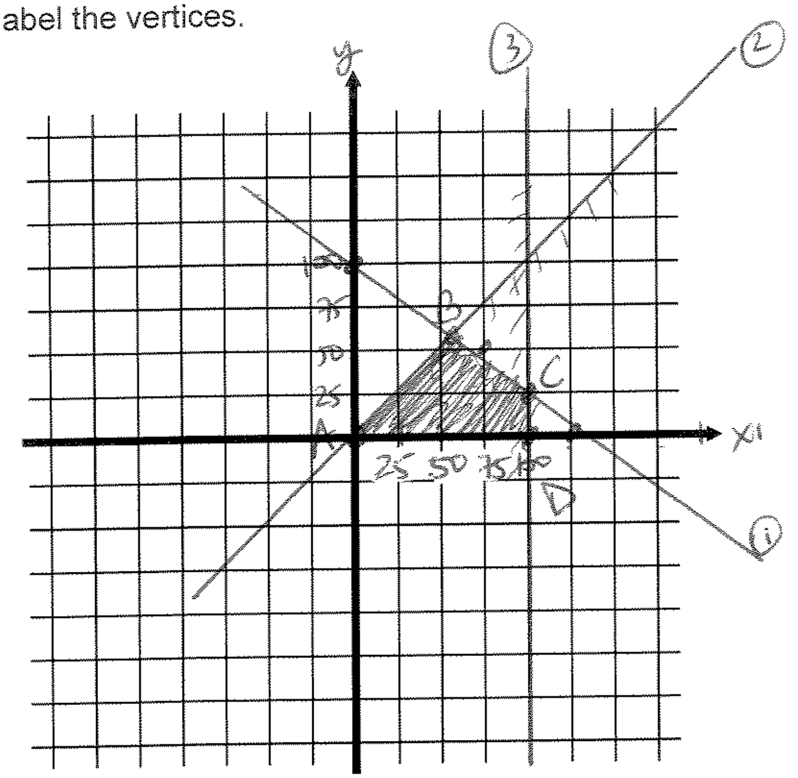
$y = -\frac{2}{3}x + 100$

$\frac{5}{3}x = 100$

$x = 100\left(\frac{3}{5}\right)$

$x = 60$

$y = 60$



A: $(0,0)$

B: $(60,60)$

C: $\left(100, \frac{100}{3}\right)$

D: $(100,0)$

C: ③ & ①

$y = -\frac{2}{3}(100) + 100$

$= \frac{100}{3}$

D: ③ + x-axis

$(100,0)$

(c) Suppose that you can model your daily profit by the function $P(x,y) = 20x + 25y$. How many pairs of each type of ski should you try to manufacture in a day in order to maximize profit?

A: $P = 0$

B: $P = 20(60) + 25(60) = 45(60) = 2700$

C: $P = 20(100) + 25\left(\frac{100}{3}\right) = 2833$ max

D: $P = 20(100) + 0 = 2000$

Number of classic ski pairs: 100 Number of skate ski pairs: ~33

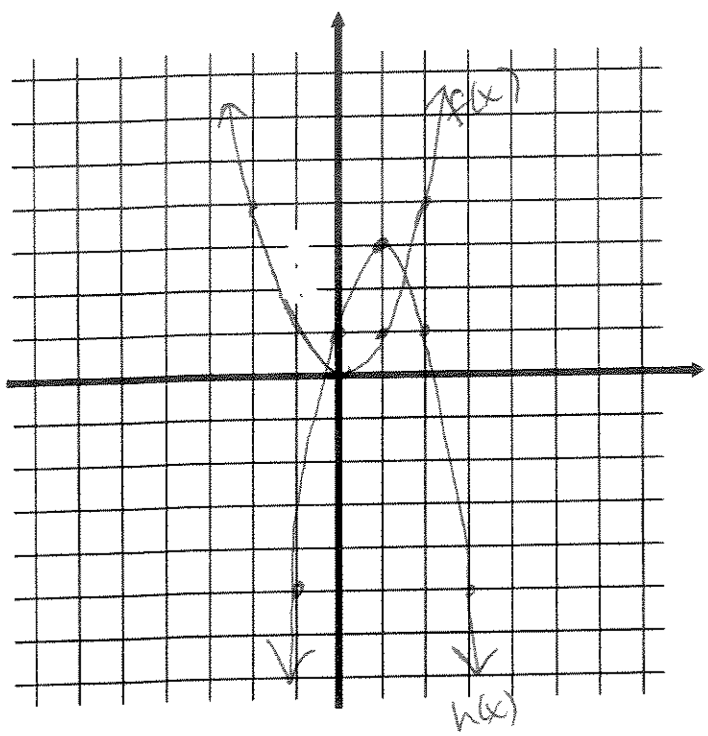
7. (a) Describe the transformations (stretches, shrinks and/or reflections) for this function $h(x) = -2(x-1)^2 + 3$. Compare it to the base graph of $f(x) = x^2$.

Shift(s) (if any): 1 to the right and 3 up

Stretch/shrink (if any): stretch vertically by 2

Reflection(s) (if any): ~~ref~~ reflect vertically (across x-axis)

(b) Below is the graph of $f(x) = x^2$. On these same axes, graph the function *h(x)* given in part (a).



8. Let $A=\begin{bmatrix} 2 & 3 \\ 9 & 7 \\ -3 & 6 \end{bmatrix}$, $B=\begin{bmatrix} -3 & 12 \\ 2 & -5 \\ 8 & 11 \end{bmatrix}$, $C=\begin{bmatrix} 8 & -7 \\ -4 & 2 \end{bmatrix}$. Compute the following, or explain why it is not possible.

$$\begin{bmatrix} 2 & 3 \\ 9 & 7 \\ -3 & 6 \end{bmatrix} + \begin{bmatrix} -6 & 24 \\ 4 & -10 \\ 16 & 22 \end{bmatrix} =$$

$$\begin{bmatrix} -4 & 27 \\ 13 & -3 \\ 13 & 28 \end{bmatrix}$$

(a) $A + 2B =$ ______________________

(b) $C - 4B =$ impossible; C & B are not the same size

$$\begin{bmatrix} -3 & 12 \\ 2 & -5 \\ 8 & 11 \end{bmatrix} \begin{bmatrix} 8 & -7 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} -24-48 & 21+24 \\ 16+20 & -14-10 \\ 64-44 & -56+22 \end{bmatrix}$$

$$\begin{bmatrix} -72 & 45 \\ 36 & -24 \\ 20 & -34 \end{bmatrix}$$

(c) $BC =$ ______________________

9. Let $f(x)=x^2+1$, $g(x)=\frac{x}{x+3}$, $h(x)=6x+7$. Find the following.

(a) Domain of g(x): $x \in \mathbb{R},\ x \neq -3$

(b) $(fh)(2)$ = 95

$f(2)h(2) = (2^2+1)(6(2)+7) = 5(12+7) = 5(19) = 95$

(c) $(f \circ h)(x)$ = $36x^2+84x+50$

$f(h(x)) = f(6x+7) = (6x+7)^2+1 = 36x^2+84x+50$

(d) $h(x) + f(x) + 3$ = $x^2+6x+11$

$(6x+7)+(x^2+1)+3 = x^2+6x+11$

(e) $g^{-1}(x)$ = $\frac{3x}{1-x}$

$x = \frac{y}{y+3}$

$x(y+3) = y$

$xy+3x = y$

$3x = y - xy$

$3x = y(1-x)$

$\frac{3x}{1-x} = y = g^{-1}(x)$

10. Jordan just bought a home. He borrowed $210,000 at a fixed rate of 5.1% interest, compounded monthly, for 25 years.
(a) What monthly payment does he make?

$S = 210000$

Amortization loan problem

$r = 0.051 \quad n = 12$

$t = 25 \quad N = 12(25) = 300$

$$R = S\left(\frac{r_c}{1-(1+r_c)^{-N}}\right)$$

$r_c = \frac{0.051}{12} = 0.00425$

$$R = 210000\left(\frac{0.00425}{1 - 1.00425^{-300}}\right) \approx \$1239.90$$

Monthly payment: $1,239.90

(b) If he decides to sell the house after making exactly 60 payments, what is his payoff for the loan?

Loan payoff:

$k = 60$

$N - k = 300 - 60 = 240$

$$S_{240} = 1239.90\left(\frac{1 - 1.00425^{-240}}{0.00425}\right)$$

$\approx \$186,313.16$

Loan payoff: $186,313.16