Business Algebra
Final Review

1. Find the inverse of the following matrix, if possible. If it's not possible, then explain why.

$$A=\begin{bmatrix}1 & -2\\ 0 & 5\end{bmatrix}$$

2. For $A=\begin{bmatrix}1 & 0 & 2\\ 3 & 2 & 1\\ 4 & 0 & 3\end{bmatrix}$, $B=\begin{bmatrix}5 & 3\\ 1 & 2\end{bmatrix}$ and $C=\begin{bmatrix}4 & 2\\ 3 & 5\end{bmatrix}$, perform the indicated matrix operations, if possible. If not possible, explain why.

(a) $A+A^T$
(b) BC

3. Use Gauss-Jordan Elimination to solve the following system.

$$2x - 4y + 2z = -4$$
$$4x - 9y + 7z = 2$$
$$-2x + 4y - 3z = 10$$

4. Given the arithmetic sequence
-2, 1, 4, 7, 10, ...

(a) Find the 100th term.
(b) Find the sum of the first 100 terms.

5. How much would have to be invested at the end of each year at 6% interest compounded annually to pay off a debt of $80,000 in 10 years?

6. A lottery prize worth $1,000,000 is awarded in payments of $10,000 five times a year for 20 years. Suppose the money is worth 20% compounded 5 times per year. What is its present value?

7. For $f(x)=\sqrt{1-x}$ and $g(x)=x^2+1$
(a) State the domain for both functions.
(b) Find $g\circ f$ and state the domain of this new function.
(c) Find $\frac{f}{g}$.

8. Solve the equation.

$$\frac{2x}{x-1}+\frac{1}{3}=\frac{5}{6}+\frac{2}{x-1}$$

9. Find the equation of the line passing through the points (-1, 1) and (2, 3).

10. Graph the linear inequality. $-4x < 6y$

11. Graph the system of inequalities and shade the solution region. Label all vertices for the solution (shaded) region.

$3x+4y \ge 12$
$x-y \ge 2$
$x \le 6$
$y \ge 0$

12. Find the maximum of the objective function $P=2x+y$ subjected to the following constraints.

$x \ge 0$
$y \ge 0$
$x+y \le 10$
$2x+3y \le 24$

13. If the cost of production for a product is given by $C(x)=x^2+11x+84$ and the revenue is given by $R(x)=30x$,
   (a) Find the profit function $P(x)$.
   (a) Find the break-even point(s).
14. If 100 feet of fence is used to fence in a rectangular yard, then the resulting area is given by $A(x)=x(50-x)$ where x feet is the width of the rectangle and $(50-x)$ feet is the length. Determine the width and length that give the maximum area.
15. Let $f(x)=-(x-1)^2+4$.
   (a) Solve $f(x)=0$ to find the x-intercepts.
   (b) Find the vertex of the parabola.
   (c) Sketch the graph, showing the vertex and x-intercepts.
16. Suppose that the population of Smalltown, USA grows according to the formula $P(t)=3200e^{0.025t}$ where time t is measured in years.
   (a) What is the initial population of the town (at $t=0$)?
   (b) How long will it take the population to double?
   (c) What is the population after one year?
17. Rewrite $\log_2 32 = x$ in exponential form and solve for x.
18. Solve for x. $|3-4x|=13$
19. The Utah Company manufactures a certain product that has a selling price of $40 per unit. Fixed costs are $1,600 and variable costs are $20 per unit. Determine the least number of units that must be sold for the company to have a profit of no less than $5,000.
20. A rectangular plot of land has an area of 18,000 square feet. If its length is five times its width, how much fencing would be required to surround the property?
21. For the following functions, find the following.

$$f(x)=\frac{x+1}{3x^2+20x+25} \qquad g(x)=-x$$

   (a) The domain of $f(x)$
   (b) The domain of $g(x)$
   (c) $f(-1)$
   (d) $f(0)$
   (e) $(f \circ g)(2)$
22. Graph the function $y=|x-2|-1$ and determine the x- and y-intercepts.
23. The students at a university buy 3,000 graphing calculators per year when they cost $50 each, and they buy 2,000 calculators per year when they cost $100 each. Let P be the price per calculator and Q be the quantity of calculators sold. Assuming the relationship between P

and Q is linear, give an equation expressing P in terms of Q.

24. Find the value for x which maximizes the quadratic function

$$f(x) = -x^2 + 11x - 24$$

25. Solve the following equations.

(a) $\ln(2x+7)=0$

(b) $e^{2x}=9$

(c) $\log x + \log 3 = 2$

26. Jeremy wants to make one savings deposit today so that in 7 years, he will have $16,000. Given an interest rate of 4% compounded semiannually (twice a year), how much money should Jeremy deposit?

27. Brittany is 25 years old and she plans to retire when she turns 60. When she retires, she would like to have $1,000,000 of savings. She is going to achieve the savings by contributing to a sinking fund between now and her retirement, with equal monthly payments paid at the end of each month. Assume the interest rate is 6% per year, compounded monthly. How much should her monthly payments be?

28. Given the matrices A and B, perform the indicated operations or state that it's not possible. If it's not possible, explain why.

$$A = \begin{bmatrix} 1 & 5 & -1 \\ 2 & 3 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

(a) AB

(b) BA

(c) B^{-1}

29. Determine if the system of equations below has any solutions. If a solution exists, find it.

$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 14 \\ 2x + y + 2z &= 10 \end{aligned}$$

30. Maximize the objective function z = 4x – 3y subject to the constraints:

$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \\ x + y &\leq 4 \\ 3x + 2y &\geq 6 \end{aligned}$$

31. Find all solutions to the following equation and inequality.

(a) $|3x+2|>20$

(b) $|-2x+6|=|3x+5|$

32. The Cordova Company sells bicycles. They pay $164 for each bicycle and have monthly fixed costs of $5500. If they sell every bicycle for $185, how many bicycles do they need to sell each month to have a profit of at least $5000?

33. Solve for x. $x^2+3=4x$

34. For the following functions, find the following.

$$f(x)=x^2-1 \quad g(x)=2x-1$$

(a) Domain of $\left(\frac{g}{f}\right)(x)$

(b) $(f \circ g)(x)$

(c) $g^{-1}(x)$

(d) $(f-g)(x)$

35. Find the x- and y-intercepts and the vertex of $f(x)=2x^2-4x-6$ algebraically. Use this

information to sketch a graph of $f(x)$.

36. A gardener has two fertilizers that contain different concentrations of nitrogen. One is 7% nitrogen and the other is 13% nitrogen. How many pounds of each should she mix together to obtain 27 pounds of 9% concentration?

37. Solve for x.

(a) $\log_2 32 = x$

(b) $27^{x-2} = 9^{3x+9}$

(c) $\log_{10} 2x + \log_{10}(x-5) = 2$

38. Suppose that $2000 is put into an account earning 6% interest compounded quarterly. How many years will it take for the balance in the account to reach $3000?

39. You are buying a $220,000 house with a down payment of $25,000. If the interest rate is 6%, compounded monthly, determine the size of the monthly payments (at the end of the month) you must make over the next 30 years to pay off the house.

40. Mr. Johnson has two debts that he needs to pay off. The first is $5000 for his student loans and it is due in 4 years. The second debt is $3000 for his purebred cat, due in 5 years. If Mr. Johnson wants to pay off both debts with a single payment in two years, how much will his payment be, assuming an interest rate of 9% compounded quarterly?

41. Given the matrices A and B, perform the indicated operations or state that it's not possible. If it's not possible, explain why.

$$A = \begin{bmatrix} 1 & 3 \\ -2 & 2 \\ 4 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 2 & 1 & -1 \\ 3 & 0 & 4 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 4 & -5 & 6 & 0 \\ 7 & 3 & 1 & 4 \end{bmatrix}$$

(a) AB

(b) BA

(c) $2B - C$

42. Follow the steps below to solve the system of linear equations using an inverse matrix.

$$2x - 5y = 2$$
$$-x + 3y = 1$$

(a) Write the system above as a matrix equation, i.e. In the form $A\begin{bmatrix} x \\ y \end{bmatrix} = b$.

(b) Find A^{-1}.

(c) Use A^{-1} to solve the system of equations.

43. Maximize the objective function $P = x + 2y$ subject to the constraints:

$$x \geq 0$$
$$y \geq 0$$
$$x + y \geq 5$$
$$x + y \leq 12$$
$$x \leq 10$$

44. Find all solutions to the following equation. $\frac{2x+5}{x+7} = 1 + \frac{x-11}{2x+14}$

45. Write the equation of the line through the point (1, 2) that is perpendicular to the line through the points (3, 2) and (-1, -6). Write the answer in slope-intercept form.

46. If $f(x) = -2x^2 - x + 4$ and $g(x) = x - 5$, find each of the following, simplifying as far as possible.

(a) $g(f(-3))$

(b) $f(g(x))$

47. Find the inverse, $f^{-1}(x)$, of the function $f(x) = \sqrt[5]{x^3 - 5} + 1$.

48. The daily profit from the sale of x units of a product is $P(x) = 10x - 0.5x^2 + 150$ dollars.

(a) Find the domain of $P(x)$.

(b) Find the x-intercept(s) and y-intercept.

(c) Find the maximum value of $P(x)$. (Hint: Use the formula for the vertex of a parabola.)

49. If the demand function for a commodity is given by the equation $p^2 + 5q = 200$ and the supply function is given by $40 - p^2 + 3q = 0$, find the equilibrium quantity and price.

50. Suppose $\ln x = 3$, $\ln y = 5$ and $\ln z = 2$. Use logarithmic properties to find $\ln\sqrt{\frac{xy}{z^2}}$.

51. Solve for x.

(a) $500 = 600 - 600e^{-0.4x}$

(b) $8 + \log(5x) = 2\log x + 7$

52. If $1,000 is invested at 2% per month, the future value S at any time *t* (in months) is given by $S = 1,000(1.02)^t$. When will the investment double?

53. A small business loan is for $50,000 for 10 years at 9% interest, compounded twice per year. The loan was taken out 6 ½ years ago. Find the payoff amount after the company makes the 14th semi-annual payment of $3843.81.

54. Grandparents decide they want to pay for their first grandchild's college education. To do this, they anticipate needing $200,000 in 18 years. If they have an investment that earns 11%, compounded monthly, how much should the grandparents deposit at the beginning of each month to reach their goal?

55. Given the matrices A, B and C, perform the indicated operations or state that it's not possible. If it's not possible, explain why.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 1 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 0 & -3 & 4 \\ 2 & -1 & 5 \\ 3 & 0 & 1 \end{bmatrix}$$

(a) $A + 3C$

(b) AB

(c) BA

56. Solve the following system of equations using any method.

$$x - 2y + z = 7$$
$$3x + y - z = 2$$
$$2x + 3y + 2z = 7$$

57. A company manufactures one type of table and one type of chair. It takes 2 hours to make a table and 6 hours to make a chair. The company has a maximum of 1800 total work hours to use in manufacturing each day, and it can deliver at most 400 total pieces (tables and chairs combined) per day.

(a) Find the equations for the constraints. Then, graph the feasible region. (Make sure to define what your variables represent.)

(b) If the company sells the table for $60 and the chair for $30, how many of each piece should it produce per day to maximize its sales?

58. Given the line $2x + y = 4$:

(a) Find the equation of the line through the point (-2, 1) that is parallel to the given line. *(Give answer in slope-intercept form.)*

(b) Find the equation of the line through the point (0, 4) that is perpendicular to the given line. *(Give answer in slope-intercept form.)*

59. Solve each equation. (Give exact answers.)

(a) $3x^2+1=-5x$

(b) $x^2-13x+36=0$

60. The supply and demand for a product are given by $2p-q=27$ and $(2p-5)q=363$, respectively. Find the equilibrium quantity and price.

61. Joe has a total of $2500 to invest into two accounts, earning simple interest. The accounts have annual interest rates of 5% and 10%. The total interest made from his investments in one year is $200.

(a) Write the system of equations that describe this scenario. Use x to represent the amount invested at 5% and y to represent the amount invested at 10%.

(b) Write the matrix equation for this system of equations.

(c) Solve the system of equations.

62. A chocolate truffle maker plans to make truffles for a street fair. She makes two kinds of truffles: a dark one that she sells for $1 a piece, and a lighter chocolate truffle that she sells for 80 cents a piece. She needs 2 teaspoons of heavy whipping cream and 10 grams of chocolate to make a dark truffle and 4 teaspoons of heavy whipping cream and 5 grams of chocolate to make a light truffle. She has a total of 1600 teaspoons of heavy whipping cream and 5000 grams of chocolate that she can use for these truffles.

(a) Find the inequalities for the constraints. Then, graph the feasible region and label the corner points. (Make sure to define what your variables represent.)

(b) How many truffles of each kind should she make to maximize revenue and what is that maximum revenue?

63. Solve for x.

(a) $\log_3(x^2)-\log_3(x+3)=\log_3(x-2)$

(b) $3^{x^2+x-3}=\frac{1}{27}$

64. Sketch the graph of the given functions in the xy-plane. Clearly label (1) two points on each graph and (2) any asymptotes.

(a) $y=3^{-2x}$

(b) $y=\log(x+1)$

65. Paul borrows $40,000 from the bank at an APR of 12% compounded monthly. If the payments on this loan are made at the end of each month, what are his payments if Paul wants to pay off his debt in 4 years? What is the total amount that Paul pays to the bank?

66. (a) Dave has a paper route that earns him $1000 per year, which he deposits, at the end of each year, in an account yielding 5% interest compounded annually. He does this for five years, from the age of 14 to 18. If after that he makes no deposits or withdrawals, but leaves the money in the account, how much money does he have when he is 65 years old?

(b) Dave's brother, Leo, wins a prize of $3,000 in a contest at age 14. He puts this in an account yielding 6% interest compounded annually. If he also makes no withdrawals or deposits, but leaves the money in the account, how much money does he have when he is 65 years old?

67. The startup costs for a street vendor are $2000, and his cost per unit to produce food is $1. Demand for his product dictates that the price at which q thousand units would be sold is $4-q$. Therefore, if q is the number of thousands of units sold, and cost and revenue are measured in thousands of dollars, the cost function is $C(q)=2+q$ and the revenue function is $R(q)=(4-q)q$.

(a) At which price(s) will the vendor break even? What quantities (in thousands) will he sell

at these prices?
(b) What is the maximum profit? How many thousand units are sold to attain this profit? What is the price?

68. Given the matrices A, B, and C, compute the following, if possible. If it's not possible, state the reason why.

$$A=\begin{bmatrix} 2 & 1 & 3 \\ 0 & 5 & -1 \end{bmatrix}, \quad B=\begin{bmatrix} 4 & 0 \\ -3 & -2 \end{bmatrix}, \quad C=\begin{bmatrix} 7 & 0 \\ 2 & 6 \\ -1 & 4 \end{bmatrix}.$$

(a) AB
(b) B^{-1}
(c) BA
(d) $3A-C^{T}$

69. Solve for x. (Show all work without a calculator.)
(a) $\ln x+\ln(x+2)=\ln 35$
(b) $5(2^{3x})-15=305$

70. Martha and Paul buy a house for \$175,000. They have a \$25,000 down payment and expect to amortize the rest of the debt with monthly payments over the next 30 years. The interest on the debt is 5.1% compounded monthly.
(a) What are the monthly payments?
(b) Find the total amount of house loan payments (excluding down payment).
(c) Find the total amount of interest paid on this loan.

71. Kilam is 20 years old. Investing in mutual funds, he can earn 8% interest, compounded quarterly.
(a) In order to have \$50,000 in the account to buy a sports car when he is 30 years old, how much money must he deposit at the end of each quarter?
(b) When Kilam turns 30, he decides not to buy the car, but leaves his money in the account (still earning compounded interest), now *withdrawing* \$2,000 at the end of each quarter. After how many quarters does he run out of money?

Answer Key:

(1) $A^{-1}=\begin{bmatrix} 1 & \frac{2}{5} \\ 0 & \frac{1}{5} \end{bmatrix}$

(2) (a) $\begin{bmatrix} 2 & 3 & 6 \\ 3 & 4 & 1 \\ 6 & 1 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 29 & 25 \\ 10 & 12 \end{bmatrix}$

(3) (-52, -28, -6)

(4) (a) 295 (b) 14,650

(5) $6,069.44

(6) $245,049.99

(7) (a) f(x) domain: $x \le 1$; g(x) domain: $x \in \boldsymbol{R}$

(b) $g(f(x))=2-x$; domain $x \in \boldsymbol{R}$

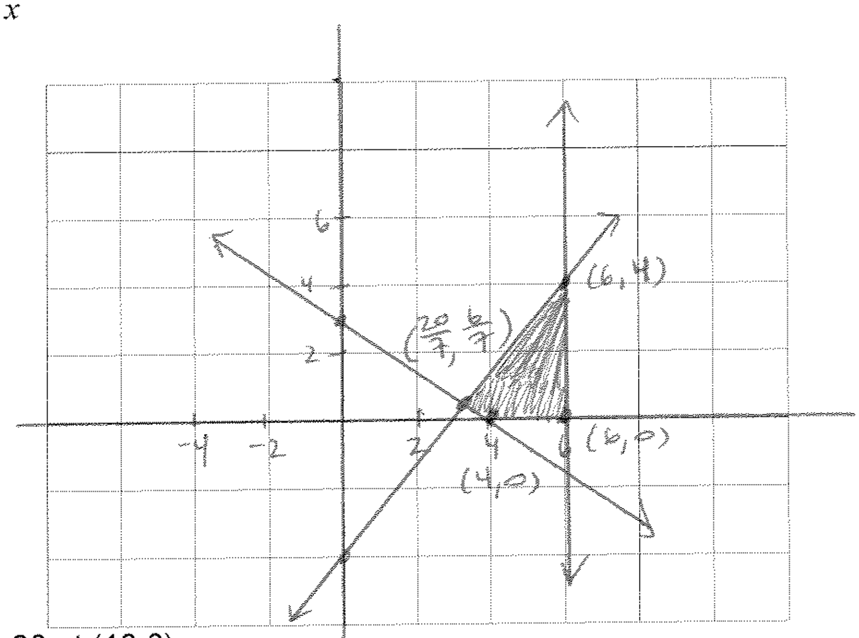
(c) $\left(\frac{f}{g}\right)(x)=\frac{\sqrt{1-x}}{x^2+1}$

(8) N.S.

(9) $y=\frac{2}{3}x+\frac{5}{3}$

(10) $y>\frac{-2}{3}x$

(11)

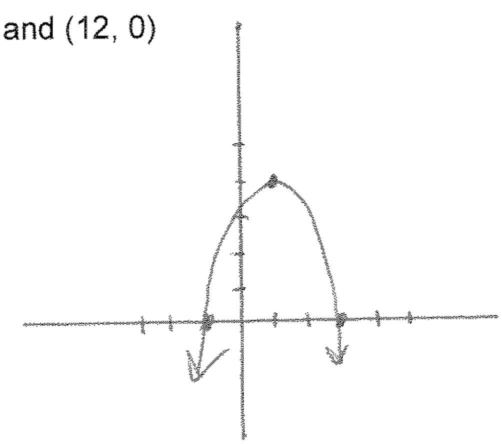


(12) Max P = 20 at (10,0)

(13) (a) $P(x)=-x^2+19x-84$ (b) (7, 0) and (12, 0)

(14) width = 25 ft, length = 25 ft

(15) (a) (3, 0) and (-1, 0) (b) (1, 4) (c)



(16) (a) 3200 people (b) $t=\frac{\ln 2}{0.025}\approx 27.7\text{ years}$ (c) 3281 people

(17) $2^x=32$; $x=5$

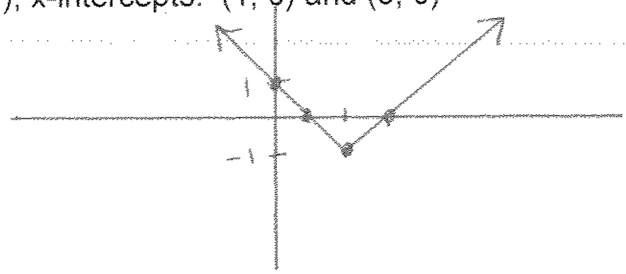
(18) x = 4, 2.5

(19) 330 units

(20) about 2276.8 feet

(21) (a) $x\in \boldsymbol{R}$, $x\neq -5$, $-\frac{5}{3}$ (b) $x\in \boldsymbol{R}$ (c) 0 (d) $\frac{1}{25}$ (e) $\frac{1}{3}$

(22) y-intercept: (0, 1), x-intercepts: (1, 0) and (3, 0)



(23) $p=-\frac{1}{20}q+200$

(24) x = 5.5

(25) (a) x = -3 (b) x = ln 3 (c) $x=\frac{100}{3}$

(26) $12,126.00

(27) $701.90

(28) (a) $\begin{bmatrix} 3 & 0 & -4 \\ 3 & 2 & -1 \end{bmatrix}$

(b) not possible; number of columns of B doesn't equal number of rows of A

(c) $\begin{bmatrix} -1 & -1 & 1 \\ 2 & 2 & -1 \\ -1 & -2 & 1 \end{bmatrix}$

(29) solution exists; (1, 2, 3)

(30) max of 16 at (4, 0)

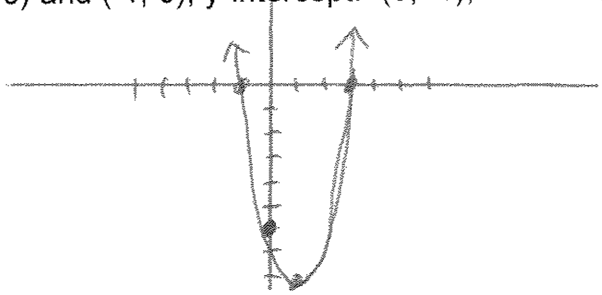
(31) (a) $x>6$ or $x<-\frac{22}{3}$ (b) $x=\frac{1}{5}$ or -11

(32) 500 bicycles

(33) x = 1, 3

(34) (a) $x\in \boldsymbol{R}$, $x\neq -1, 1$ (b) $4x^2-4x$ (c) ½(x-1) (d) x^2-2x

(35) x-intercepts: (3, 0) and (-1, 0); y-intercept: (0, -6); vertex: (1, -8)



(36) 18 lbs of 7% nitrogen fertilizer and 9 lbs of 13% nitrogen fertilizer

(37) (a) x = 5 (b) x = -8 (c) x = 10

(38) about 6.8 years

(39) \$1,169.12

(40) \$6,481.69

(41) (a) $\begin{bmatrix} 9 & 3 & 13 & 5 \\ 6 & -4 & 6 & 6 \\ 3 & 8 & 8 & -2 \end{bmatrix}$

(b) not possible; number of columns of B is not equal to number of rows of A

(c) $\begin{bmatrix} -4 & 9 & -8 & -2 \\ -1 & -3 & 7 & 0 \end{bmatrix}$

(42) (a) $\begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ (c) (11, 4)

(43) max at (0, 12) with P = 24

(44) N.S.

(45) $y=-\frac{1}{2}x+\frac{5}{2}$

(46) (a) -16 (b) $-2x^2+19x-41$

(47) $f^{-1}(x)=\sqrt[3]{(x-1)^5+5}$

(48) (a) $x\geq 0$ (b) x-intercepts: (-10, 0) and (30, 0); y-intercept: (0, 150) (c) \$200

(49) q = 20, p = \$10

(50) 2

(51) (a) $x\approx 4.4794$ (b) x = 50

(52) in about 35 months

(53) \$19,825.88

(54) \$294.07

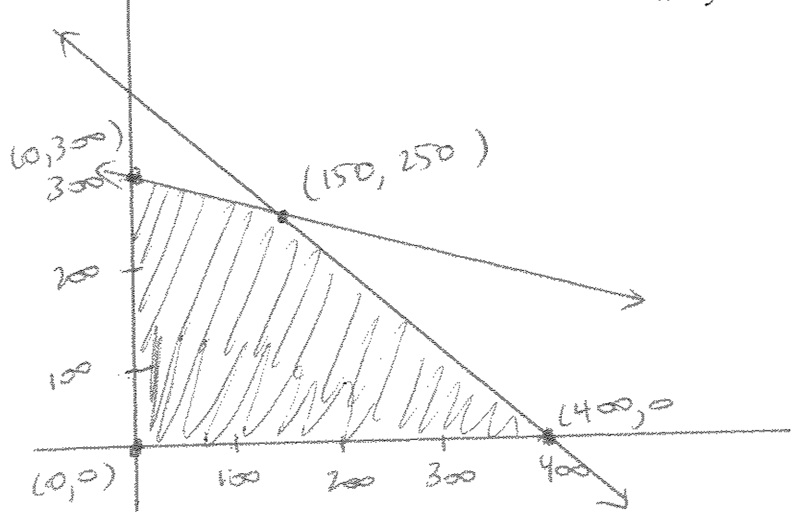
(55) (a) $\begin{bmatrix} 1 & -7 & 15 \\ 7 & -2 & 17 \\ 12 & 1 & 7 \end{bmatrix}$

(b) not possible; number of columns of A not equal to number of rows of B

(c) $\begin{bmatrix} 6 & 8 & 20 \\ 5 & 4 & 9 \end{bmatrix}$

(56) (2, -1, 3)

(57) (a) x = number of tables, y = number of chairs; $x\geq 0\ ,\ y\geq 0$, $2x+6y\leq 1800$, $x+y\leq 400$



(b) 400 tables and 0 chairs

(58) (a) y = -2x – 3 (b) $y=-\frac{1}{2}x+4$

(59) (a) $x=\frac{-5\pm\sqrt{13}}{6}$ (b) x = 4, 9

(60) q = 11, p = $19

(61) (a) 0.05x + 0.1y = 200 and x + y = 2500

(b) $\begin{bmatrix}0.05 & 0.1\\ 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}200\\ 2500\end{bmatrix}$

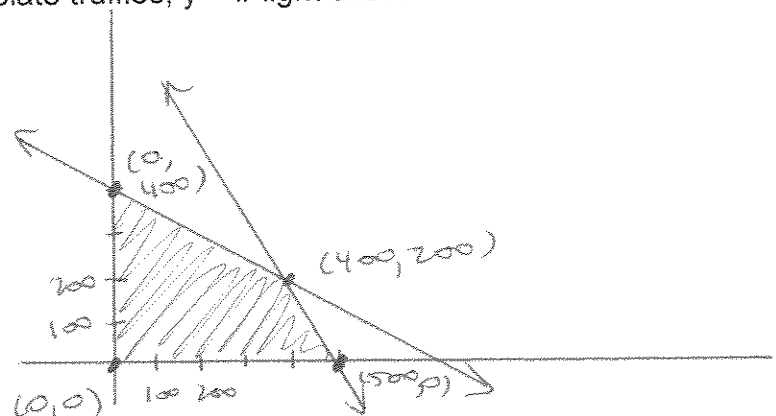
(c) x = $1000, y = $1500

(62) (a) x = # dark chocolate truffles, y = # light chocolate truffles

$x\geq 0\ ,\ y\geq 0$

$2x+4y\leq 1600$

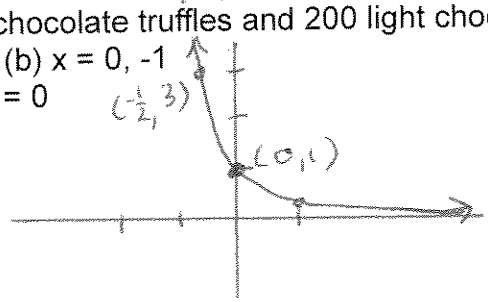
$10x+5y\leq 5000$



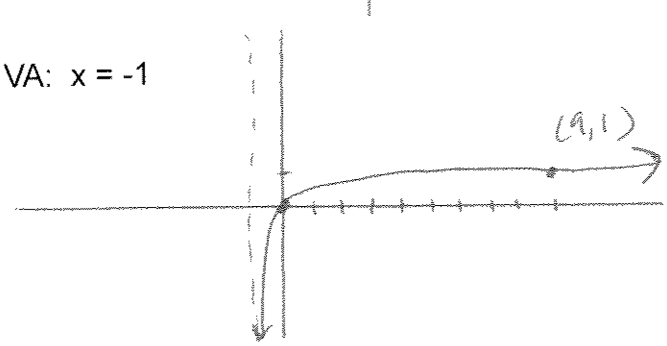
(b) 400 dark chocolate truffles and 200 light chocolate truffles for $560 revenue

(63) (a) x = 6 (b) x = 0, -1

(64) (a) HA: y = 0



(b) VA: x = -1



(65) $1,053.35; $50,560.96

(66) (a) $54,736.73 (b) $58,576.09

(67) (a) $2 and $3; 2000 units and 1000 units

(b) max profit is $250 for 1500 units at a price of $2.50 per unit

(68) (a) not possible; number of columns of A is not equal to number of rows of B

(b) $\begin{bmatrix} \frac{1}{4} & 0 \\ -\frac{3}{8} & -\frac{1}{2} \end{bmatrix}$ (c) $\begin{bmatrix} 8 & 4 & 12 \\ -6 & -13 & -7 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 1 & 10 \\ 0 & 9 & -7 \end{bmatrix}$

(69) (a) x = 5 (b) x = 2

(70) (a) $814.42 (b) $293,192.88 (c) $143,192.88

(71) (a) $827.79 (b) after 35 quarters (or 8.75 years)