1.7 Graphical Linear Programming

Ex1 Find min & max values of objective fn $f = 4x + 3y$ on feasible region given by

$$\begin{cases} 2x+3y \le 12 \\ 4x-2y \le 8 \\ x \ge 0,\ y \ge 0 \end{cases}$$

feasible region ⇒

closed & bounded ⇒

optimal solution ⇒

★ optimal soln occurs at "corners"

★ if no corners, may be no optimum values

1.7 (cont)

Ex 2 minimize $g = 22x - 17y$ subject to

constraints
$$\begin{cases} 8x + 5y \geq 100 \\ 12x + 25y \geq 360 \\ x \geq 0, y \geq 0 \end{cases}$$

1.7 (cont)

Ex3 A contractor builds two types of homes. The Carolina requires one lot, $160,000 capital, and 160 worker-days of labor. The Savannah requires one lot, $240,000 capital, and 160 worker-days of labor. The contractor owns 300 lots and has $48,000,000 available capital and 43,200 worker-days of labor. The profit on the Carolina is $40,000 and on the Savannah, it's $50,000. How many of each type of home should be built to maximize profit? What is max profit?

2.1 Basic operations w/ Matrices

Defns

$A = B$

A^T

O matrix

Square matrix

Column or row vector

Vocab

matrix ⇒

entry ⇒

scalar ⇒

order (size) ⇒

Ex 1 For $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 0 & -2 \\ 6 & 1 & 5 \end{bmatrix}$

(a) size =

(b) a_{13} =

(c) A^T =

(d) first column vector =

2.1 (cont)

Ex 2 Given $A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ -5 & 1 & 0 & 1 \\ 3 & -2 & 7 & 0 \end{bmatrix}$

(a) what size (order) is A?

(b) what is a_{24}? a_{31}?

(c) write zero matrix same size as A.

(d) Find A^T.

(e) write $-A$.

2.1 (cont)

Ex 3 Given $A = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 4 & 2 & 1 & 5 \\ -1 & 0 & -2 & 0 \end{bmatrix}$

$B = \begin{bmatrix} 2 & 2 & 5 & 1 \\ 0 & 0 & -4 & -3 \\ 1 & 4 & -1 & 2 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ -1 & 0 & 3 \\ 4 & 5 & 0 \end{bmatrix}$

(a) Find $2A + B$

(b) $A - 3C^T$

matrix Addition

$A + B$

Scalar Multiplication

cA

2.1 (cont)

Ex 4 Given $A = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$ $B = [2 \ 9 \ 1]$

$C = [-3 \ 1 \ 5]$

$D = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$

Find

(a) $B^T + D$

(b) $B - (A - D)^T$

(c) $(2C + A^T)^T$

2.2 Matrix Multiplication

Defn ① matrix multiplication AB

Given A (size $m \times n$) and B (size $n \times p$), AB is an $m \times p$ matrix w/ ij entry given by

$$a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{in}b_{nj}$$

i.e. the product/sum of the i^{th} row of A w/ the j^{th} column of B)

② identity matrix I ⇒ always a square matrix; has 1 in each diagonal entry and zeros everywhere else

★ closest we have to mult. identity for matrices

$$IA = AI = A$$

$$I = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$

Properties of Matrix Multiplication

① $(AB)C = A(BC)$ Associativity

② $A(B+C) = AB + AC$ > Right & left Distributivity

③ $(B+C)A = BA + CA$

④ $(AB)^T = B^T A^T$

WARNING $AB \neq BA$

2.2 (cont)

Ex 1 Given $A = \begin{bmatrix} 1 & 0 & 4 \\ 5 & 1 & 2 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 2 \\ 3 & -2 \\ -1 & 0 \end{bmatrix}$

Find AB and BA, if possible.

Ex 2 Is $(AA^T)^T = A^T A$?

2.2 (cont)

Ex 3 Given $A = \begin{bmatrix} 0 & 1 & 3 \\ 2 & 0 & 1 \\ 0 & 0 & -4 \end{bmatrix}$, find A^2.

Ex 4 (#32) Solve for X.

$$\begin{bmatrix} -8 & -2 \\ -6 & 2 \end{bmatrix} X = \begin{bmatrix} -50 & 2 & -8 \\ -6 & -30 & 22 \end{bmatrix}$$

Ex 5 Solve for x.

$$\left(\begin{bmatrix}-5\\10\\-19\end{bmatrix}+\begin{bmatrix}-2x\\13\\-8\end{bmatrix}\right)+\begin{bmatrix}-8\\-4\\-7\end{bmatrix}=\begin{bmatrix}-5\\19\\34\end{bmatrix}$$