2.3 Gauss-Jordan Elimination

Vocab

Augmented matrix: A matrix that represents a system of linear eqns.

elementary row operations:

1. switch two rows

2. multiply any row by a nonzero constant

3. replace one row w/ the result of adding it w/ a nonzero multiple of another row.

Gauss-Jordan elimination: process for solving a system of linear eqns, using elementary row ops, until we have triangular matrix.

like this
$$\left[\begin{array}{ccc|c} 1 & 3 & 4 & 5 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 1 & -4 \end{array}\right]$$

this is augmented matrix for

$$\begin{aligned} x + 3y + 4z &= 5 \\ y + 2z &= 7 \\ z &= -4 \end{aligned}$$

2.3 (cont)

Ex1 Solve.

$$10x+y=6$$
$$3x+y+2z=3$$
$$2x-y-2z=2$$

Ex2 $-2x+y=1$
$2x-y=7$

2.3 (cont)

Ex 3 Solve.

$$\begin{aligned} 3x-2y-7z&=0 \\ x-y-z&=1 \\ -x+2y-3z&=-4 \end{aligned}$$

Ex 4

$$\begin{aligned} 3x-y&=3 \\ x+z&=3 \\ 2x-y-z&=3 \end{aligned}$$

2.3 (cont)

Ex 5 Solve

$$\begin{aligned} x+y+z&=1 \\ x-y-z&=1 \\ -x+y-z&=1 \end{aligned}$$

2.4 Inverse Matrices

<u>Defn</u> A^{-1} read "A inverse" (it's <u>not</u> an exponent)

$$A^{-1} \cdot A = I = A \cdot A^{-1}$$

- A^{-1} can only exist for square matrix A $(n \times n)$ and is also square $n \times n$.

<u>EX 1</u> Find A^{-1} for

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Method to find A^{-1} for A

1. If A is not square, A^{-1} DNE.
2. If A is square,
   (a) augment A with identity matrix
   (b) Perform elementary row ops on augmented matrix until the left side is I
   (c) what's on the right side is A^{-1}.

2.4 (cont)

Ex 2 Find A^{-1}.

(a) $A = \begin{bmatrix} 7 & 6 \\ 2/3 & 4 \end{bmatrix}$

(b) $A = \begin{bmatrix} 6 & 0 & 5 \\ -3 & 5 & -3 \\ 7 & 3 & 6 \end{bmatrix}$

Ex 3 Use A^{-1} from Ex2(b) to solve

$$\begin{aligned} 6x \quad\quad + 5z &= 1 \\ -3x + 5y - 3z &= 0 \\ 7x + 3y + 6z &= 7 \end{aligned}$$

To solve

$$Ax = b$$

(where A is $n \times n$ matrix, x is $n \times 1$ column vector of variables and b is $n \times 1$ column vector of constants), we can left-multiply both sides by A^{-1}.

$$A^{-1}Ax = A^{-1}b$$

$$Ix = A^{-1}b$$

$$x = A^{-1}b$$