3.2 Functions

Relation	vs.	Function
defined by a set of ordered pairs (x, y)		a relation such that every input has exactly one output

domain ⇒ set of allowable inputs

range ⇒ set of outputs

Ex 1 Relation or Function?

(a) x = person y = car owned by that person

(b) x = person y = their kid

(c) x = person y = their mom

(d) x = student y = grade in Math1090 class.

Vertical Line Test

If we graph all the ordered pairs (of a relation) on a Cartesian coordinate system, and every vertical line goes through the graph at most one time, then it's a function.

Ex 2 Are these functions? Identify the domain.

(a) $y = f(x) = 6x^2$

(b) $y^2 = 4x^2$

Ex 3 Evaluate, given $f(x) = 4x^2 - 5x$

(a) $f(-2)$

(b) $f(2)$

(c) find the domain

(d) find the range

3.2 (cont)

Ex 4 Evaluate for $f(x) = \frac{x+3}{x-2}$

(c) find domain

(a) $f(1)$

(b) $f\left(\frac{2w+1}{w-1}\right)$

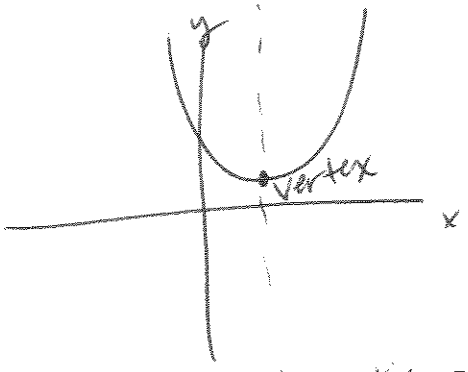
Ex 5 find domain

(a) $f(x) = \sqrt{2x-1}$

(b) $g(x) = \frac{3}{x^2-25}$

3.3 Parabolas: Quadratic Equations in Two Variables

Quadratic Fn $\Rightarrow$ $y = f(x) = ax^2 + bx + c$
(a quadratic eqn in two variables)
when we graph all the solutions to this, the points form a parabola.

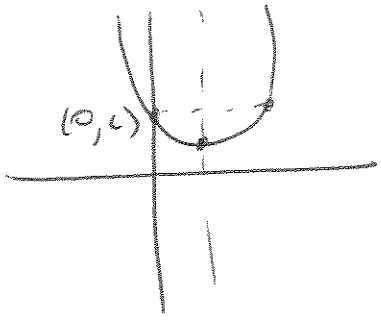


↦ axis of symmetry

For $y = ax^2 + bx + c$,
if $a > 0$, $\cup$ concave up
if $a < 0$, $\cap$ concave down

Let's figure out where the vertex is (algebraically) so we can always find it.

if we plug in $x = 0$, we get pt on y-axis $\Rightarrow$
$x = 0 \Rightarrow y = a(0^2) + b(0) + c \Leftrightarrow y = c$, i.e. parabola goes thru $(0, c)$



We can see, by symmetry of parabola, that there is another pt whose y-value is c.

3.3 (cont)

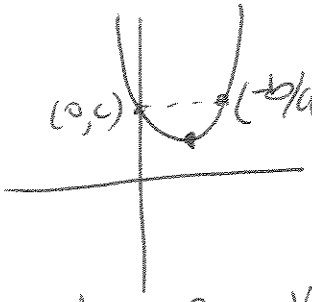
$c = ax^2+bx+c \iff 0 = ax^2+bx$

$0 = x(ax+b)$

$x=0$, or $ax+b=0$

$ax=-b$

$x=-b/a$



You can see (from symmetry) that x-value of vertex is halfway between 0 and $-b/a$, i.e. $x=-b/2a$.

$\Rightarrow$ **vertex at $\left(-b/2a, f(-b/2a)\right)$**

axis of symmetry: $x=-b/2a$

Ex 1 For $y=-2x^2-4x+6$

(a) find vertex

(b) Is the vertex a min or max pt?

Ex 2 For $y = x^2 - 6x + 9$,

(a) find vertex.
(b) Is it a min or max pt?
(c) find zeros of graph.
(d) Find the axis of symmetry.
(e) sketch graph.

3.3 (cont)

Ex 3 If 100 ft of fencing is used to enclose a rectangular yard, find the area function. Find the dimensions of the rectangle that maximize area.

3.4 Quadratic Business Applications

Supply Demand + Market Equilibrium

Ex 1 If the supply function for a commodity is $p = q^2 + 8q + 20$ and the demand function is $p = 100 - 4q - q^2$, find the equilibrium quantity and equilibrium price. (Sketch both curves.)

3.4 (cont)

Ex 2 For the last example, if an $8 tax is placed on production & passed through the supplier, find the new equilibrium pt.

3.4 (cont)

Break-Even Pts and Maximization

Ex 3 If a company has total costs $C(x) = 1600 + 1500x$ and total revenue is $R(x) = (1600 - x)x$, find the break even pts.

Break Even pts occur when

$R(x) = C(x)$

$\iff P(x) = 0$

3.4 (cont)

Ex 4 Find maximum revenue given $R(x) = 1600x - x^2$.

Ex 5 Suppose a company has fixed costs of \$300 and variable costs of $\frac{3}{4}x + 1460$ dollars per unit, where x = total # units produced. Suppose further that its selling price is $1500 - \frac{1}{4}x$ dollars per unit.

(a) Find break even pts.

3,4 (cont)

Ex 5 (cont)

(b) Find max revenue

(c) Find max profit, and price that yields it.