3.7 Transformations of Graphs

☆ see table page 182 in book

Transformations base graph $f(x)$

shift	reflection	stretch/shrink
$h(x) = f(x) \pm c$	$g(x) = -f(x)$	$k(x) = cf(x)$ $c \in \mathbb{R}$
$h(x) = f(x \pm c)$	$g(x) = f(-x)$	$k(x) = f(cx)$ $c \in \mathbb{R}$

Ex 1 Describe transformation of $f(x) = -(x-2)^2 + 3$ compared to base graph of $y = x^2$.
Sketch graph of $f(x)$.

3.7 (cont)

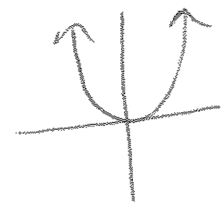
Ex 2 Describe x-form and sketch graph.

(a) $g(x) = -2|x-3|+1$

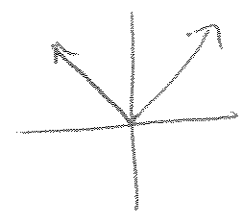
(b) $h(x) = 4(x+2)^3 - 3$

Base graphs

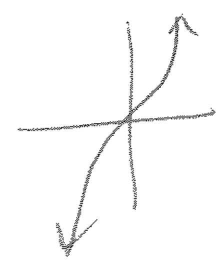
$y = x^2$



$y = |x|$



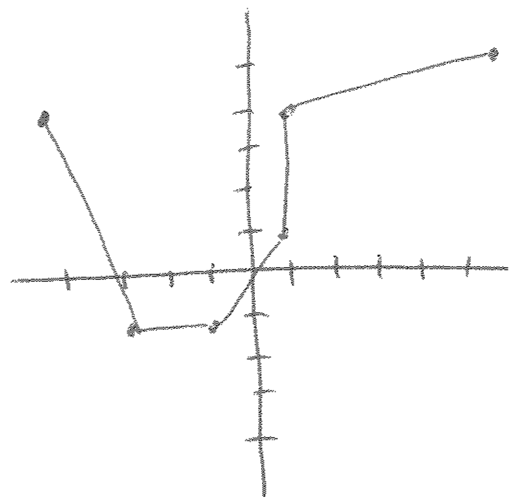
$y = x^3$



$y = \sqrt{x}$

3.7 (cont)

Ex 3 Given this graph, sketch the indicated transformed graph.



(a) $f(-x)+1$

(b) $f(x+1)$

(c) $-f(x)+1$

3.8 Combination of Functions &

4.1 Inverse Functions

Fns

- Addition $(f+g)(x) = f(x) + g(x)$
- Subtraction $(f-g)(x) = f(x) - g(x)$
- Multiplication $(fg)(x) = f(x)\,g(x)$
- Division $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ (as long as $g(x) \neq 0$)
- Composition $(f \circ g)(x) = f(g(x))$ Read "f of g of x"

Ex 1 Given $f(x) = 2x+5$ $g(x) = \frac{1}{x^3}$, find

(a) $(f \circ g)(x)$

(b) $(f+g)(1)$

(d) $\left(\frac{f}{g}\right)(x)$

(c) $(g \circ f)(1)$

Ex 2 Given $f(x) = x^2 - 1$ $g(x) = \frac{x}{2}$ $h(x) = \sqrt{x-1}$, find

(a) $(h \circ f)(x)$

(b) $(g - h)(1)$

(c) $(hf)(3)$

(d) $g(h(x))$

(e) $h(f(g(x)))$

Inverse Fns

An inverse fn basically "undoes" what the original fn did to the input x

notation: $f^{-1}(x)$ read "f inverse of x"

$$\boxed{f^{-1}(f(x)) = f(f^{-1}(x)) = x}$$

Ex3 Are $f(x) = 5x-1$ and $g(x) = \frac{x+1}{5}$ inverse fns?

Does every fn have an inverse? No!

A fn that has an inverse passes the horizontal line test (w/ its graph)

one-to-one: every input has exactly one output and every output has exactly one input.

3.8 & 4.1 (cont)

☆ Inverse fn graph is mirror image of original fn across line $y=x$

Ex 4

Does $y=x^2$ have an inverse fn?
Can we restrict its domain so it does have an inverse?

Ex 5 Find inverse of $f(x)=4(x-1)^3$

Strategy to find inverse

① "Pants" Technique

② Standard Technique

3.8 & 4.1 (cont)

Ex 6 Find inverse of $f(x) = \sqrt[3]{\dfrac{x+1}{2x+3}}$

Ex 7 Are $f(x) = 2\sqrt{x} - 1$ and $g(x) = \frac{1}{4}(x+1)^2$
inverses of each other?