

Zero is symbolized by an oval shape in the Hindu-Arabic numeration system. It represents the lack of a quantity or measurement. It begins the set of whole numbers and separates positive numbers from negative numbers. Sometimes zero is implied. This occurs when it is unnecessary to place a zero in front of a number to represent the place value prior to the number. For example, in 47 there is no zero representing the hundreds, thousands, tens of thousands, etc. (zeros representing the hundreds, thousands, and tens of thousands place value would appear like this “00047”). This is implied because if one were to place zeros ahead of the value, one would be expected to draw an infinite number of zeros.

We need a zero because it is vital in maintaining place value when there are no quantities for that specific place value. Thus, when there are no 1s in a number, we need zero to illustrate this. 90 is much different from 9 (9 would be the number displayed if there was no zero). Another example of this is the absence of tens in the number 608. If there were no zero to hold the place of the tens, the number would look like this, “68.” Because, again, 68 and 608 are very different numbers, without zero our lives would be much more difficult. Zero is also important because it plays the role of additive identity. When we need to change the form of a problem or manipulate it in order to be able to work with it more easily, we can add zero. This will change the form of the quantity but not the value. Thus, $5+0=5$. This can also be applied when working with more difficult problems such as $27+36$. When we add zero to this problem it can become easier to work with. The form of zero we will add to this problem will be $3-3=0$.

$$27+36=27+36+3-3=27+3+36-3=(27+3)+36-3=30+36-3=(30+36)-3=66-3=63$$

This is where we are adding zero. We rearranged the problem using the commutative property. We used the associative property. We followed the order of operations. We used the associative property. We followed the order of operations.



That problem was easier when we added zero rather than simply relying on addition. It uses strategies that can be completed in one’s head (which is termed “mental math”) instead of writing the problem out and completing it on paper. This generally yields quicker, more practical results.

Zero behaves differently with different operations. The following examples and descriptions explain how zero will behave in the situation of the specified operation.

With **addition**, zero behaves as an identity. The problem would result in the same amount if zero were absent. The following example illustrates this:

$$7+0=7$$

If zero is removed from the problem, it becomes $7=7$. Notice that the answers remain the same in both problems. This is the identity function of zero. This is also true when the order of the numbers in the problem is reversed (using the commutative property). Thus, $0+7=7$, $7+0=7$, and $7=7$.

Unlike addition, zero does not behave as an identity in **subtraction** because there is no commutative property in subtraction. However, when a zero is subtracted from a whole number, the difference is the original whole number (which makes it seem to act like an identity). See the following examples:

$$2-0=2$$

$$2=2$$

$$98-0=98$$

$$98=98$$

Zero is not an identity because when a number is subtracted from zero (this would appear with the number order reversed from the previous arrangement) the result will not be the original whole number. It will be a negative number, and the integer will appear the same as the original non-zero whole number with a negative sign in front of it. Thus, when two is subtracted from zero the difference is negative two (or $0-2= -2$). The following examples illustrate this:

$$0-98= -98$$

$$0-5,639= -5,639$$

If a number is **multiplied** by zero, the result will always be zero. Though multiplication has an identity property, the identity in multiplication is 1 rather than zero.

$$89,435 \times 0=0$$

$$4 \times 0=0$$

When we use the commutative property to reverse the order of the numbers in the multiplication problem, the result is still zero. Thus, $0 \times 89,435=0$ and $89,435 \times 0=0$.

This makes sense if we think of multiplication as repeated addition. If we have no sets of 4 four times, we still have 0. This can also be considered using concrete examples. If there are no cookies to begin with, and we add the absence of cookies four times, we will still have no cookies. Thus, $0 \times 4=0$.

Zero behaves differently with **division**. When zero is divided by any non-zero whole number, the result will be zero.

$$0/3=0$$

$$0/88,527=0$$

When any whole number is divided by zero, the result is undefined. This is because when the problem is rewritten into a multiplication problem (which is possible since division is the opposite of multiplication), there is no possible whole number to solve the problem. This is shown in the following example:

$$3/0=a$$

$$3=0 \times a$$

There is no possible solution for a because when zero is multiplied by any whole number, the result will always be zero. Thus, the problem is undefined.

When zero is divided by zero, the result is also undefined. This is because when we rewrite the problem as a multiplication problem, there are infinitely many possible solutions for the solution.

$$\begin{aligned} 0/0 &= c \\ 0 \times c &= 0 \end{aligned}$$

Notice that c could be any whole number because any whole number multiplied by zero will produce zero.

As an **exponent**, zero will cause the answer to be one if the base of the exponent is a whole number other than zero.

$$\begin{aligned} 9^0 &= 1 \\ 783^0 &= 1 \\ 0^0 &= \text{undefined} \end{aligned}$$

When zero is the base of an exponent and the exponent is any whole number other than zero, the result will always be zero.

$$\begin{aligned} 0^1 &= 0 \\ 0^9 &= 0 \\ 0^0 &= \text{undefined} \end{aligned}$$

When zero is the base and the exponent, the result is undefined. This is because of discrepancies in the patterns. If 0^0 were to follow the pattern of zero as an exponent, the result should be 1 (thus, $0^0=1$ which is untrue). However, if 0^0 were to follow the pattern of zero as the base, the result should be 0 (thus, $0^0=0$ which is untrue). This means that there should be two distinctly different solutions to the same problem and because of this inconsistency, $0^0=\text{undefined}$. These patterns are illustrated in the following problems:

Pattern 1 (zero as an exponent)	Pattern 2 (zero as the base)
$5^0=1$	$0^5=0$
$4^0=1$	$0^4=0$
$3^0=1$	$0^3=0$
$2^0=1$	$0^2=0$
$1^0=1$	$0^1=0$
$0^0=?$	$0^0=?$

Overall, zero is a very important element of our number system and has many unique properties. It behaves differently with various operations. It can simplify our problems and was a key development in the advancement of human civilizations.