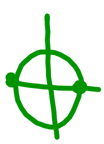
① $\tan^2 x = -\sqrt{3}\tan x$ $\quad x \in [0, 2\pi)$

$\tan^2 x + \sqrt{3}\tan x = 0$

$\tan x\left(\tan x + \sqrt{3}\right) = 0$

$\tan x = 0$ or $\tan x = -\sqrt{3}$

$\boxed{x = 0, \pi}$



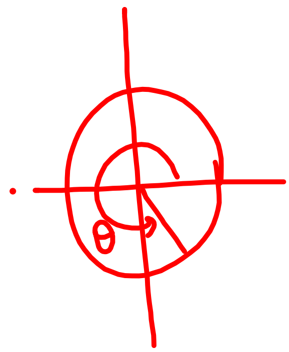
$= \dfrac{-\sqrt{3}/2}{1/2}$ $\Big\}$

$\sin x = -\sqrt{3}/2$ and $\cos x = 1/2$

or $\dfrac{\sqrt{3}/2}{-1/2}$ $\Big\}$ $\sin x = \sqrt{3}/2$ and $\cos x = -1/2$

$\boxed{x = \frac{2\pi}{3} \text{ or } \frac{5\pi}{3}}$

(2) 2θ is in what quadrant; θ is in Q4.



$$\frac{3\pi}{2} \le \theta \le 2\pi$$

$$2\left(\frac{3\pi}{2}\right) \le 2\theta \le 2(2\pi)$$

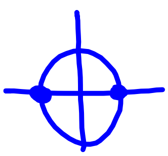
$$3\pi \le 2\theta \le 4\pi$$

$\Rightarrow 2\theta$ is in Q3 or Q4.

(3) $\sin(2x) + 2\cos x \sin(2x) = 0$

$\sin(2x)\left(1 + 2\cos x\right) = 0$

$\sin(2x) = 0$ or $1 + 2\cos x = 0$

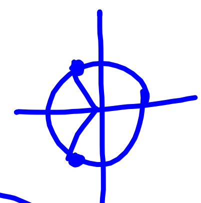


$2x = 0, \pi$

$2x = n\pi, \; n \in \mathbb{Z}$

$\boxed{x = \frac{n\pi}{2}, \; n \in \mathbb{Z}}$

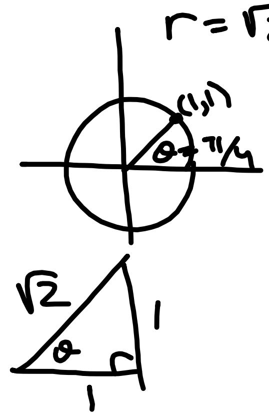
$\cos x = -1/2$



$\boxed{x = \begin{cases} \frac{2\pi}{3} + 2n\pi \\ \frac{4\pi}{3} + 2n\pi \end{cases}} \; n \in \mathbb{Z}$

if $x \in [0, 2\pi)$, then $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \frac{4\pi}{3}, \frac{2\pi}{3}$

(9) $(1+i)^{10} = \left[\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]^{10}$



$$= (\sqrt{2})^{10}\left(\cos\left(\frac{10\pi}{4}\right) + i\sin\left(\frac{10\pi}{4}\right)\right)$$

$$= \left((\sqrt{2})^2\right)^5\left(\cos\left(\frac{5\pi}{2}\right) + i\sin\left(\frac{5\pi}{2}\right)\right)$$

$$= 32(0 + i(1)) = \boxed{32i}$$

$a + bi$

$r = \sqrt{a^2+b^2}$, $\tan\theta = \frac{b}{a}$

(5) Simplify.

$$2\cos^2(22.5^\circ)-1$$

$$\cos(22.5^\circ)=\cos\left(\frac{45^\circ}{2}\right)=\pm\sqrt{\frac{1+\cos(45^\circ)}{2}}=\sqrt{\frac{1+\sqrt{2}/2}{2}}$$

$$=\sqrt{\frac{1}{2}+\frac{\sqrt{2}}{4}}$$

$$=\sqrt{\frac{2+\sqrt{2}}{4}}$$

$$=\frac{\sqrt{2+\sqrt{2}}}{2}$$

$$2\cos^2(22.5^\circ)-1$$

$$=2\left(\frac{2+\sqrt{2}}{4}\right)-1$$

$$=\frac{1}{2}(2+\sqrt{2})-1=\sqrt{2}/2$$

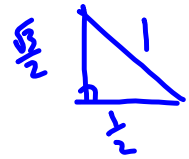
double angle

$$2\cos^2\theta-1=\cos(2\theta)$$

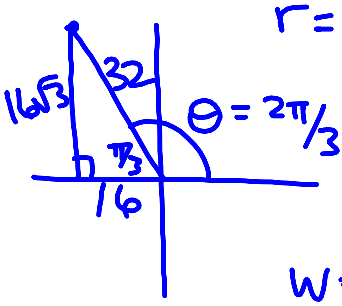
$$2\cos^2(22.5^\circ)-1$$

$$=\cos(2(22.5^\circ))=\frac{\sqrt{2}}{2}$$

(6) $w = \left(\left(-16 + 16\sqrt{3}\,i\right)^3\right)^{1/5}$



$r = \sqrt{16^2 + 16^2\sqrt{3}^2} \quad \cdots \quad = 32$



$w = \left(\left(32\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)\right)^3\right)^{1/5}$

$w = \left(32^3\left(\cos\left(3\left(\frac{2\pi}{3}\right)\right) + i\sin\left(3\left(\frac{2\pi}{3}\right)\right)\right)\right)^{1/5}$

$= \left(32^3\left(1 + 0i\right)\right)^{1/5}$

$= \left(32^3\left(\cos(0) + i\sin(0)\right)\right)^{1/5}$

$= \boxed{32^{3/5}\left(\cos\left(\frac{2\pi n}{5}\right) + i\sin\left(\frac{2\pi n}{5}\right)\right) \quad n = 0, 1, 2, 3, 4}$

$$= \begin{cases} 32^{3/5}(1) & n = 0 \\ 32^{3/5}\left(\cos\left(\frac{2\pi}{5}\right) + i\sin\left(\frac{2\pi}{5}\right)\right) & n = 1 \\ 32^{3/5}\left(\cos\left(\frac{4\pi}{5}\right) + i\sin\left(\frac{4\pi}{5}\right)\right) \\ 32^{3/5}\left(\cos\left(\frac{6\pi}{5}\right) + i\sin\left(\frac{6\pi}{5}\right)\right) \\ 32^{3/5}\left(\cos\left(\frac{8\pi}{5}\right) + i\sin\left(\frac{8\pi}{5}\right)\right) \end{cases}$$

replace $32^{3/5} = 8$

$32^{3/5}$
$= (2^5)^{3/5}$
$= 8$

7

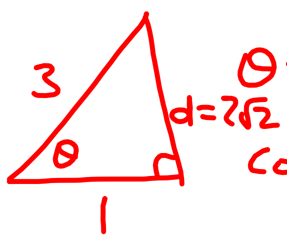
$$\sin\left(\arccos\left(\tfrac{1}{3}\right) + \arcsin\left(\tfrac{1}{2}\right)\right)$$

need addition identity

$$\sin(\theta+\beta) = \sin\theta\cos\beta + \cos\theta\sin\beta$$

$$= \sin\left(\arccos\left(\tfrac{1}{3}\right)\right)\cos\left(\arcsin\left(\tfrac{1}{2}\right)\right) + \cos\left(\arccos\left(\tfrac{1}{3}\right)\right)\sin\left(\sin^{-1}\left(\tfrac{1}{2}\right)\right)$$

$$= \sin\left(\arccos\left(\tfrac{1}{3}\right)\right)\cos\left(\pi/6\right) + \tfrac{1}{3}\left(\tfrac{1}{2}\right)$$



$\theta = \arccos\left(\frac{1}{3}\right)$

$\cos\theta = \frac{1}{3}$

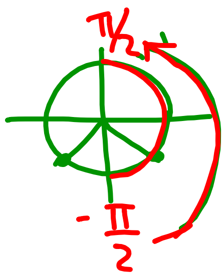
$1 + d^2 = 9 \Rightarrow d^2 = 8$

$$= \frac{2\sqrt{2}}{3}\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{6}$$

$$= \frac{2\sqrt{6}+1}{6}$$

8

$$\arcsin\left(\sin\left(\frac{5\pi}{4}\right)\right)$$



$$= \arcsin\left(\frac{-\sqrt{2}}{2}\right)$$

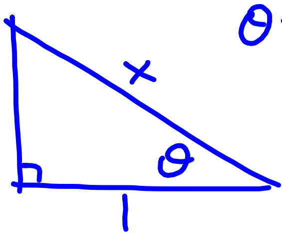
$$= \frac{-\pi}{4}$$

$$\arctan\left(\tan\left(\frac{\pi}{12}\right)\right) = \frac{\pi}{12}$$

$$\arccos\left(\cos\left(72^\circ\right)\right) = 72^\circ$$

9

$\operatorname{arcsec} x = \arccos\left(\frac{1}{x}\right)$



$\theta = \operatorname{arcsec} x$

$\sec\theta = x$

$\cos\theta = \frac{1}{x}$

$\theta = \arccos\left(\frac{1}{x}\right)$

$\csc^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right)$

$$\cot^{-1}(x) = \begin{cases} \tan^{-1}\left(\frac{1}{x}\right) & x \in Q1 \\ \tan^{-1}\left(\frac{1}{x}\right) + \pi & x \in Q2 \end{cases}$$