M1100

4.4 #9 $f(x) = \dfrac{2}{1+3e^{-2x}} = \dfrac{2}{1+3e^{-2x}}\left(\dfrac{e^{2x}}{e^{2x}}\right)$

$$f(x) = \frac{2e^{2x}}{e^{2x}+3}$$

$$f'(x) = \frac{(e^{2x}+3)(2e^{2x}(2)) - 2e^{2x}(e^{2x}(2))}{(e^{2x}+3)^2}$$

$$= \frac{4e^{4x} + 12e^{2x} - 4e^{4x}}{(e^{2x}+3)^2} = \frac{12e^{2x}}{(e^{2x}+3)^2} = 0$$

(note: the denominator is never zero)

$\Leftrightarrow \quad 12e^{2x} = 0$

but this has no solution since e^{2x} is always positive

$\Rightarrow f'(x) > 0$ for all x

$$f''(x) = \frac{(e^{2x}+3)^2(12e^{2x}(2)) - 12e^{2x}(2(e^{2x}+3)(2e^{2x}))}{(e^{2x}+3)^4}$$

$$= \frac{(e^{2x}+3)\left[(e^{2x}+3)(24e^{2x}) - 48e^{4x}\right]}{(e^{2x}+3)^4 3}$$

$$= \frac{24e^{4x} + 72e^{2x} - 48e^{4x}}{(e^{2x}+3)^3} = \frac{72e^{2x} - 24e^{4x}}{(e^{2x}+3)^3}$$

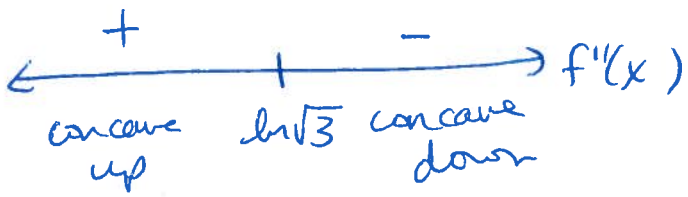
$$= \frac{24e^{2x}(3 - e^{2x})}{(e^{2x}+3)^3} = 0$$

when $3 - e^{2x} = 0$

$\Rightarrow e^{2x} = 3$

$2x = \ln 3$

$x = \frac{1}{2}\ln 3 = \ln\sqrt{3}$



$$f''(x) = \frac{24e^{2x}(3-e^{2x})}{(e^{2x}+3)^3}$$

note: $24e^{2x} > 0$ for all x

$(e^{2x}+3)^3 > 0$ for all x

inflection pt:

$(\ln\sqrt{3}, 1)$

$\ln\sqrt{3} \approx 0.55$

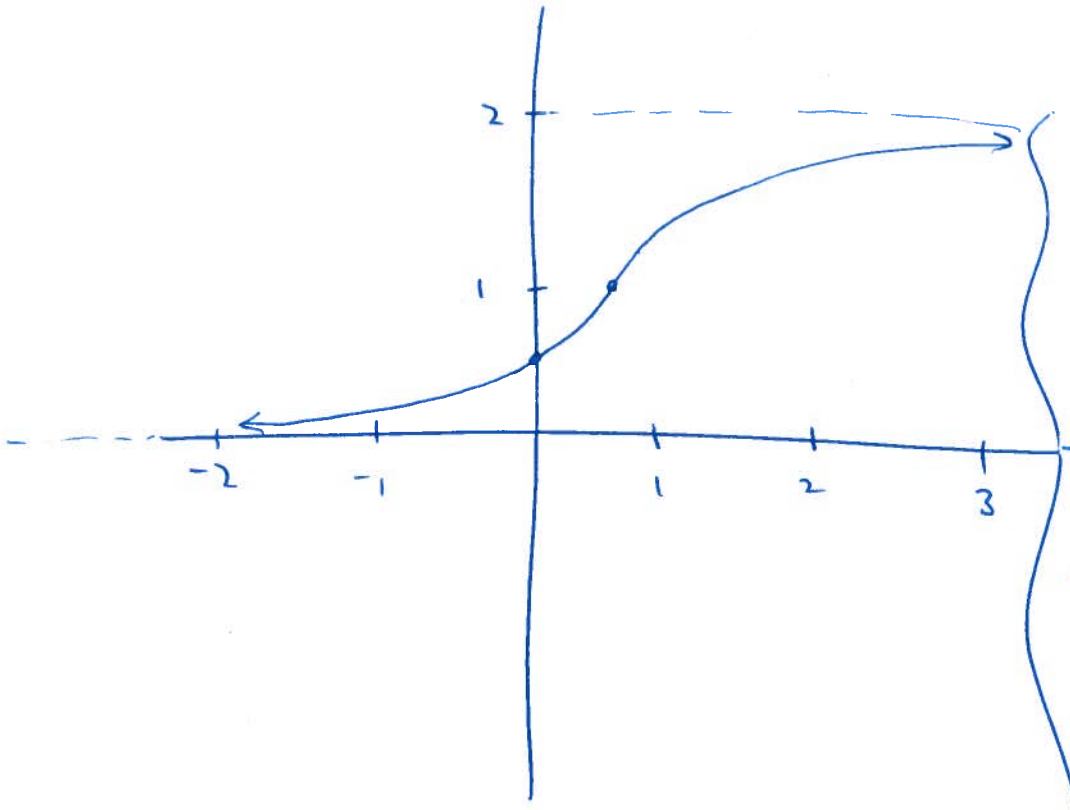
no max

no min

$$f(\ln\sqrt{3}) = \frac{2}{1+3e^{-2(\ln\sqrt{3})}}$$

$$= \frac{2}{1+3e^{\ln(1/3)}}$$

$$= \frac{2}{1+3(\frac{1}{3})} = \frac{2}{1+1} = 1$$



other point

$(0, 1/2)$

$f(0) = \frac{2}{1+3(1)} = \frac{1}{2}$

(y-intercept)

HA

$\lim_{x\to\infty} \frac{2e^{2x}}{e^{2x}+3}$

$= \lim_{x\to\infty} \frac{2e^{2x}}{e^{2x}}$

$= \lim_{x\to\infty} 2$

$\Rightarrow y=2$ is right horiz. asymptote

$\lim_{x\to-\infty} \frac{2}{1+3e^{-2x}} = 0$

$\Rightarrow y=0$ is left HA