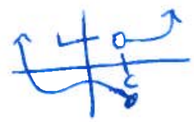
1.6 One-Sided Limits & Continuity

One-Sided Limits If $f(x) \to L$ as $x \to c$ from the left $(x<c)$, we write $\lim_{x \to c^-} f(x) = L$.

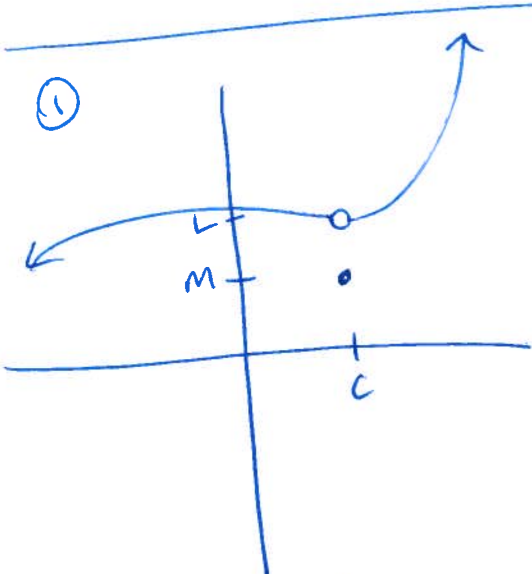
Likewise if $f(x) \to L$ as $x \to c$ from the right $(x<c)$, we write $\lim_{x \to c^+} f(x) = L$.



$\lim_{x \to c} f(x)$ exists and is L **iff** $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L$

Continuity $f(x)$ is continuous at $x=c$ if (1) $f(c)$ is defined, (2) $\lim_{x \to c} f(x)$ exists, **and** (3) $\lim_{x \to c} f(x) = f(c)$

1



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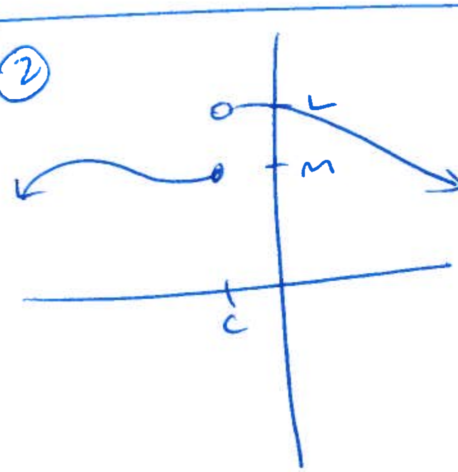

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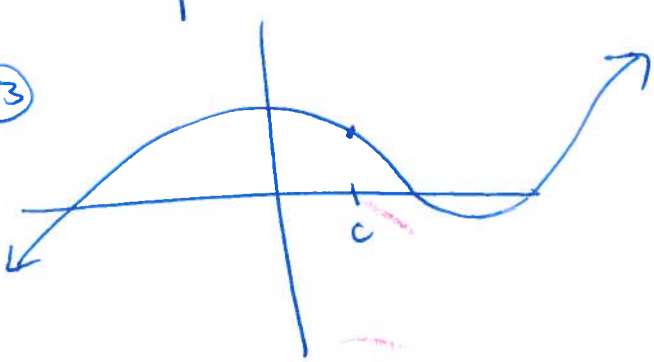

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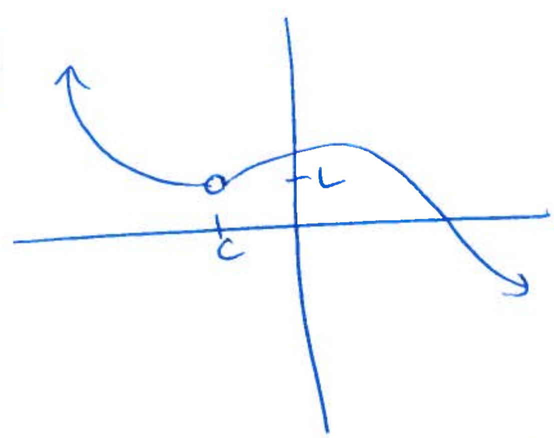


1.6 (cont)

Ex 1 Find the limits.

(a) $\lim\limits_{x \to 1^-} \dfrac{x - \sqrt{x}}{x-1}$

(b) $\lim\limits_{x \to -1^-} f(x)$ and $\lim\limits_{x \to -1^+} f(x)$ for $f(x) = \begin{cases} \dfrac{1}{x-1}, & x < -1 \\ x^2 + 2x, & x \geq -1 \end{cases}$

Is this fn continuous?

1.6 (cont)

Ex 2 Is $f(x) = \frac{\sqrt{x} - 3}{x - 9}$ continuous at $x = 3$?
at $x = 9$?
at $x = 1$?

Ex 3 Where are these fns continuous and discontinuous?

(a) $f(x) = \frac{2x}{(x+4)(x-1)}$

(b) $f(x) = \frac{x^2 - 1}{x + 1}$

1.6 (cont)

Ex 4 (#44) On 1/1/12, Sam started working for Acme Corp w/ an annual salary of \$48,000 paid each month on the last day of that month. On July 1, he received a commission of \$2,000 for his work + on Sept. 1, his base salary was raised to \$54,000/yr. Finally, on 12/21/12, he received a Christmas bonus of 1% of his base salary.

(a) sketch graph of Sam's cumulative earnings E as a fn of t. (t represents days of the year)

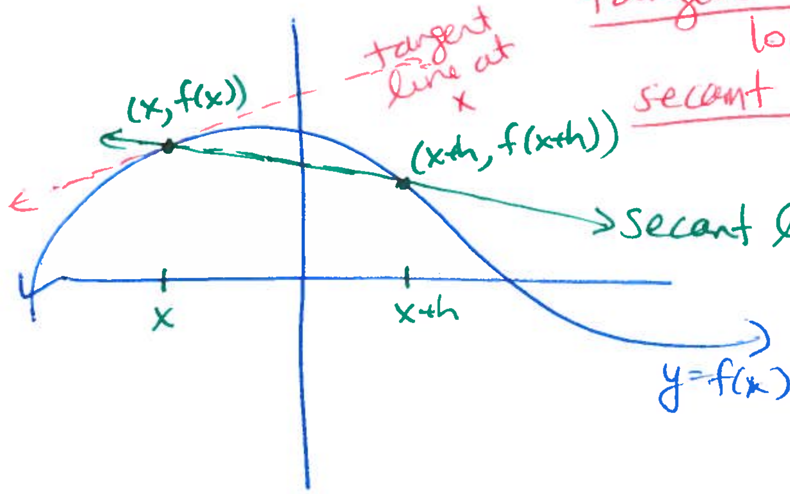
(b) where is E(t) discontinuous?

2.1 The Derivative

The derivative is a fn that represents the slope of the curve at any given x-value.

tangent line: touches the curve locally at only one pt.

secant line: goes thru curve in 2 pts.



slope of secant line

$$m = \frac{f(x+h) - f(x)}{x+h-x}$$

$$\Leftrightarrow \quad m = \frac{f(x+h) - f(x)}{h}$$

keep making h small until it's almost zero, then we get a tangent line

So slope of tangent line is slope of secant line such that h is almost zero,

i.e. $\boxed{m_{tangent} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}} = f'(x)$ (called the derivative of f(x))

slope = rate of change
= velocity
= speed
= derivative

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2.1 (cont)

Ex 1 Compute the derivative for $f(x)$. And find the slope at the given x-value.

(a) $f(x) = 2 - 7x$ at $x = -5$

(b) $f(x) = 2x^2 - 5$ at $x = 1$

2.1 (cont)

Ex 1 (cont)

(c) $f(x) = 1/x^2$ at $x=2$

(d) $y = \frac{1}{2-x}$ at $x = -3$

2.1 (cont)

Ex 2 Compute $f'(x)$ for $f(x) = \frac{1}{\sqrt{x}}$ and then find eqn of tangent line when $x = 1$.

Ex 3 Let $s(t) = \sqrt{t}$. Find the avg rate of change of $s(t)$ wrt t as t changes from $t=1$ to $t=1/4$.

Then find instantaneous rate of change at $t=1$.

2.2 Techniques of Differentiation

Notation: $D_x(y)$, $\frac{dy}{dx}$, y' , $y'(x)$

Rules

① $D_x(k) = 0$, $k \in \mathbb{R}$ (k is a constant)

② **Power Rule** $D_x(x^n) = nx^{n-1}$, $n \in \mathbb{R}$.

③ $D_x(cf(x)) = c\,D_x(f(x))$

④ $D_x(f(x) \pm g(x)) = D_x(f(x)) \pm D_x(g(x))$

(different notation: $\frac{d(f+g)}{dx} = \frac{df}{dx} + \frac{dg}{dx}$

or $(f+g)' = f' + g'$)

Ex 1 Differentiate (or find the derivative)

(a) $y = -2x + 10$

(b) $y = 2\sqrt[4]{x^3}$

2.2 (cont)

Ex 2 Compute the derivative.

(a) $y = \frac{-x^2}{25} + \frac{2}{x} - x^{2/3} + \frac{1}{4x^3} + \frac{x}{5}$

(b) $y = x^2(x^3 - 6x + 7)$

(c) $y = \frac{-7}{x^{2.1}} + \frac{5}{x^{-2.1}}$

2.2 (cont)

Ex 3 Find eqn of tangent line to $f(x) = 2x^4 - \sqrt{x} + \frac{3}{x}$ at $x = 1$.

Ex 4 Find the rate of change of $f(x)$ wrt x at $x = 1$, $f(x) = \frac{2}{x} - x\sqrt{x}$

2.2 (Cont)

Ex 5 Find relative rate of change of $f(x) = x + \frac{1}{x}$ when $x = 1$.

Relative Rate of change of $Q(x)$

$$= \frac{Q'(x)}{Q(x)}$$

Percent rate of change of $Q(x)$ wrt x

$$= \frac{100\, Q'(x)}{Q(x)}$$

Note: if $f'(c) > 0$, then the slope is positive, so the fn is increasing at $x = c$.

Likewise, if $f'(c) < 0$, then the slope is negative, so the fn is decreasing at $x = c$.