2.4 The Chain Rule

If $y = f(u)$ is differentiable (wrt u) & $u = g(x)$ is also differentiable (wrt x), then $y = f(g(x))$ is also differentiable and

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Ex 1 Find $\frac{dy}{dx}$.

(a) $y = \frac{1}{\sqrt{u}}$, $u = x^2 - 9$

(b) $y = u^5 - 3u^2 + 6u - 5$, $u = x^2 - 1$ for $x = 1$

2.4 (cont)

Ex 2 Find $\frac{dy}{dx}$.

(a) $y = f(x) = \sqrt[3]{5x^6 - 12}$

(b) $g(x) = y = \frac{(x+1)^5}{(3x-2)^3}$

(c) $y = k(x) = \frac{1-5x^2}{\sqrt{2x^3+1}}$

2.4 (cont)

Ex 3 Find eqn of tangent line to $f(x) = x^2\sqrt{2x+3}$ at $x = -1$.

Ex 4 Find second derivative of

$y = (1-2x^3)^4$

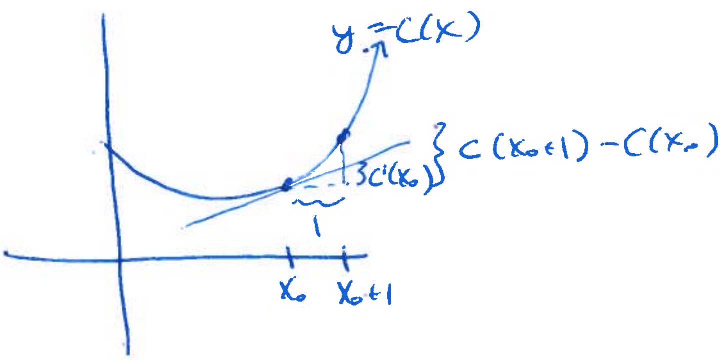
2.4 (cont)

Ex 5 Find $h'(0)$ if $h(x) = \left(\frac{g(x) - x}{3 + g(x)}\right)^2$ where $g(0) = 3$, $g'(0) = -2$.

2.5 Marginal Analysis & Approximations Using Increments

<u>marginal cost:</u> If $C(x)$ is the total cost of producing x units, then marginal cost of producing x_0 units is $C'(x_0)$.

(If x_0 is large, $C'(x_0) \simeq C(x_0+1) - C(x_0)$ = cost of producing one more unit more than x_0.)



Marginal Revenue $= R'(x_0)$

marginal Profit $= P'(x_0)$

where

$$P(x) = R(x) - C(x)$$

profit = revenue − cost

<u>Ex 1</u> If Cost is given as $C(x) = \frac{2}{7}x^2 + 65$ and $p(x) = \frac{12+2x}{3+x}$ is the unit price (price where all x products will be sold) (in dollars), (a) find revenue $R(x)$, (b) Find marginal cost (MC) and marginal revenue (MR), (c) use MC to estimate cost of producing the 21st unit & use MR to estimate revenue from sale of 21st unit, (d) what is actual cost & revenue of producing/selling 21st unit?

2.5 (cont)

Ex 1 (cont)

Approximations by Increments

If $f(x)$ is differentiable at $x = x_0$ and Δx is small change in x then

$$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x$$

$$\Leftrightarrow \quad \Delta f \approx f'(x_0)\Delta x$$

(where $\Delta f = f(x_0 + \Delta x) - f(x_0)$)

Differentials

The differential of x is $dx = \Delta x$.

And for $y = f(x)$ (a differentiable fn)

$$dy = f'(x)\,dx$$

2.5 (cont)

Ex 2 Estimate how much $f(x)=\frac{x}{x+1}-3$ will change as x decreases from 4 to 3.8.

Ex 3 An efficiency study of the morning shift at a certain factory indicates that an average worker arriving on the job at 8 a.m. will have assembled $f(x)=-x^3+6x^2+15x$ units x hours later. Approximately how many units will the worker assemble between 9 and 9:15 a.m.?