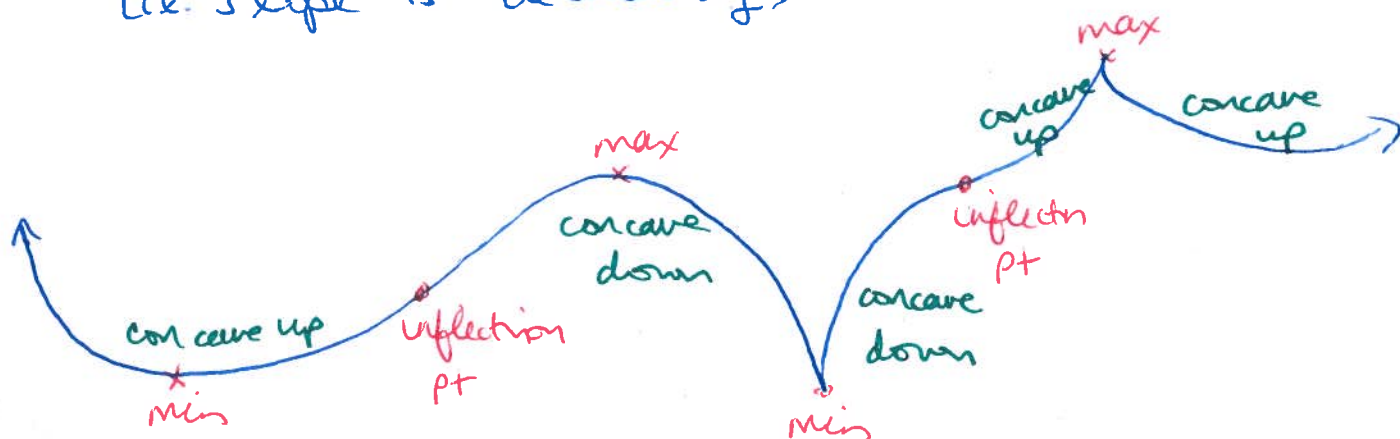


## 3.2 Concavity and Points of Inflection

$f(x)$  is concave up on  $a < x < b$  if  $f''(x) > 0$  on that interval  
(i.e. slope is increasing)

$f(x)$  is concave down on  $a < x < b$  if  $f''(x) < 0$  on that interval  
(i.e. slope is decreasing)



inflection pt pt where concavity changes (on continuous graph)  
(we can look for inflection pts where  $f''(x) = 0$  or is undefined)

### 2<sup>nd</sup> derivative test

Suppose  $f''(x)$  exists on some neighborhood around  $x=c$ ,  
and  $f'(c) = 0$ .

- or
- (1) Then if  $f''(c) > 0$ ,  $(c, f(c))$  is min pt.
  - (2) Then if  $f''(c) < 0$ ,  $(c, f(c))$  is max pt.

But if  $f''(c) = 0$  or is undefined, then the test is inconclusive

## 3.2 (cont)

Ex1 Determine where this fn is concave up/down  
+ find inflection pts.

$$f(x) = x(x+3)^2$$

Ex2 Find all min/max pts, inflection pts, first + second  
derivative sign lines + sketch graph.

(a)  $f(x) = (x^2 - 3)^3$

3.2 (cont)

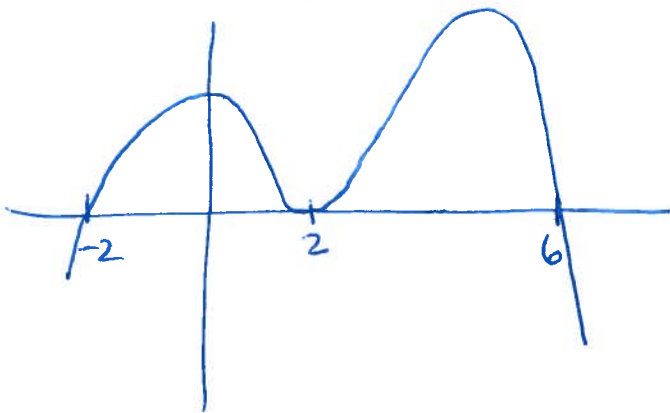
Ex2 (cont)

$$(b) \quad h(x) = \frac{(x+3)^3}{(x-1)^2}$$

### 3.2 (cont)

EX3 If  $f'(x) = x(1-x)$  (a) find intervals where  $f$  is increasing/decreasing, (b) find where graph is concave up/down, (c) find all extreme pts + inflection pts, (d) sketch graph.

EX4 This is graph of  $f'(x)$ . Sketch possible graph of  $y=f(x)$ .



## 3.3 Curve Sketching

### Asymptotes

#### VA (vertical asymptote)

$x=c$  is VA if

$$\lim_{x \rightarrow c^-} f(x) = \pm \infty$$

or  $\lim_{x \rightarrow c^+} f(x) = \pm \infty$

#### HA (horizontal asymptote)

$y=b$  is HA if

$$\lim_{x \rightarrow \pm \infty} f(x) = b$$

VA are restrictions

HA are description of  
long term behavior  
of graph

EX1 Sketch graph.

$$f(x) = \frac{x+3}{x-5}$$

To graph a fn:

- ① graph HA & VA
- ② plot intercepts
- ③ do  $f'(x)$  sign line  
and  $f''(x)$  sign line  
to find min/max  
pts and inflection pts

3.3 (cont)

Ex 2 sketch graph

$$f(x) = \frac{1}{\sqrt{1-x^2}}$$

Ex 3 Given this information, sketch possible graphs of  $f(x)$ .



### 3.3 (cont)

Ex 4 Given  $f'(x) = x^2(x+1)^3$ , find  $f''(x)$  and  $f'''(x)$   
sign lines, and  $x$ -values for inflection pts.

Ex 5 Production follows this fn over time,  
 $P(t) = \frac{t}{(t+1)^2}$  (a) sketch graph of  $P(t)$ .  
(b) what is long term behavior?