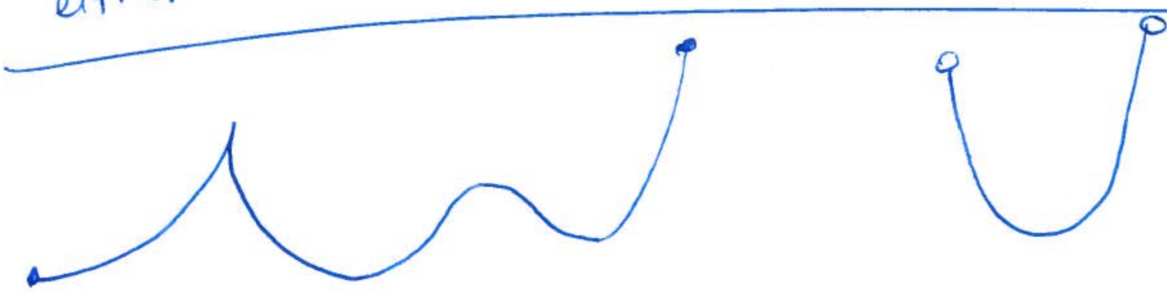
3.4 Optimization; Elasticity of Demand

Absolute max/min

if f is fn on an open interval I that contains $x=c$.
Then $f(c)$ is abs. min. if $f(c) \le f(x)$ for all x in I
and $f(c)$ is abs. max if $f(c) \ge f(x)$ " "

If ① $f(x)$ is continuous on ② closed interval $a \le x \le b$, then it has an absolute min and max pt at either a critical pt or an end pt.



Second derivative test

$f(x)$ continuous; $f'(c)=0$ at $x=c$

then $f''(c) > 0 \Rightarrow f(c)$ min value
$f''(c) < 0 \Rightarrow f(c)$ max value

3.4 (cont)

Ex 1 Find absolute min/max, on given interval.

(a) $g(x) = \dfrac{1}{x^2-9}$; $0 \le x \le 2$

(b) $f(x) = \dfrac{x^2}{x-1}$; $-2 \le x \le \frac{-1}{2}$

3.4 (cont)

Ex 2 $\quad p =$ price $\quad q = \#$ units

$C(q) =$ total cost
$R(q) =$ revenue
$P(q) =$ profit $= R - C$

$p(q) = 37 - 2q \qquad C(q) = 3q^2 + 5q + 75$

(a) Find $R(q)$, $P(q)$, $R'(q)$, $C'(q)$. Sketch graphs of P, R', C' on same axes.

For what q is P maximized?

(b) Find $A(q) = \frac{C(q)}{q}$ (average cost). Sketch A + C' on same axes.

For what q is A minimum?

4.3 Differentiation of Exponential & Logarithmic Fns

$D_x(e^x) = e^x$	$D_x(b^x) = (\ln b)\, b^x$, $b>0$, $b\neq 1$
$D_x(\ln x) = \frac{1}{x}$, $x>0$	$D_x(\log_b x) = \frac{1}{x \ln b}$, $x>0$, $b>0, b\neq 1$

Ex 1 Find derivative.

(a) $f(x) = \sqrt{1+e^x}$

(b) $h(x) = \dfrac{e^{-2x}}{x^{2/3}}$

(c) $y = x^2 3^{x^2}$

(d) $y = \ln(e^{-x}+x)$

4.3 (cont)

Ex 2 Find eqn of tangent line to $y = (x+2)e^{-3x}$

at $x=0$

Ex 3 Find $f''(x)$ for $f(x) = x^2 e^{-x}$

4.3 (cont)

Ex 4 Use logarithmic differentiation to find $f'(x)$ for $f(x) = \dfrac{e^{-3x}\sqrt{2x-5}}{(6-5x)^4}$

Ex 5 If cost is $C(x) = x^2 + 10xe^{-x}$, find $C'(x)$ (marginal cost) and what x produces minimum average cost A? $\left(A(x) = \dfrac{C(x)}{x}\right)$

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