

Math5700  
Homework #13

Given  $a_n$ , find the first three terms of the sequence  $\{a_n\}$  and find  $\lim_{n \rightarrow \infty} a_n$ , if it exists, or state that the sequence diverges.

1.  $a_n = (-1)^{n+1} \left( \frac{1}{n} \right)$

2.  $a_n = \frac{5n}{e^{2n}}$

3.  $a_n = \frac{\cos^2 n}{3^n}$

4.  $a_n = e^{-n} \ln(n)$

5.  $a_n = \frac{n^2}{2n-1} - \frac{n^2}{2n+1}$

Determine if each series is absolutely convergent, conditionally convergent, or divergent. Justify your answers.

6.  $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$

7.  $\sum_{n=1}^{\infty} (-1)^n e^{-n}$

8.  $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{3^n}{n^2+4}$

9.  $\sum_{n=1}^{\infty} \frac{2^{n-1}}{5^n(n+1)}$

10.  $\sum_{n=1}^{\infty} \frac{5^n}{n(3^{n+1})}$

11.  $\sum_{n=2}^{\infty} \frac{(-1)^n 3}{n^4-1}$

12.  $\sum_{n=1}^{\infty} \frac{10-2^n}{n!}$

13.  $\sum_{n=1}^{\infty} \frac{\sin(\sqrt{n})}{\sqrt{n^3+1}}$

14.  $1 + \frac{3}{2!} + \frac{3(5)}{3!} + \frac{3(5)(7)}{4!} + \dots$

15.  $\sum_{i=1}^{\infty} (-1)^i \frac{2i+1}{i^2+i^3}$

16.  $\frac{2}{1+e} + \frac{2}{8+e^2} + \frac{2}{27+e^3} + \frac{2}{64+e^4} + \dots$

17.  $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^5}$

18.  $\sum_{n=1}^{\infty} (-1)^n 3^{\frac{1}{n}}$
19.  $\sum_{j=1}^{\infty} \frac{(2j)^j}{(5j+3j^{-1})^j}$
20.  $\sum_{n=1}^{\infty} \frac{\cos(\frac{n\pi}{6})}{n^2}$
21.  $\sum_{i=1}^{\infty} (i^2+9)(-2)^{-i}$
22.  $\frac{1+\cos(1)}{1+1} + \frac{2+\cos(2)}{8+1} + \frac{3+\cos(3)}{27+1} + \frac{4+\cos(4)}{64+1} + \dots$
23.  $\sum_{n=1}^{\infty} \frac{e^n}{n^e}$
24.  $\sum_{k=0}^{\infty} \frac{1}{2 + \left(\frac{1}{2}\right)^k}$
25.  $\sum_{n=1}^{\infty} \frac{1-\cos n}{n^2}$
26.  $\sum_{n=5}^{\infty} \frac{1}{n^2-4n}$
27.  $\sum_{n=1}^{\infty} n^{-2} e^{\frac{1}{n}}$

Find the convergence set for these power series.

28.  $\sum_{n=0}^{\infty} \frac{(n+1)x^n}{(-3)^n}$
29.  $\sum_{n=1}^{\infty} \frac{(x+10)^n}{n(2^n)}$
30.  $\sum_{n=1}^{\infty} \frac{(-1)^n (4^{2n})(x^n)}{\sqrt{n+1}}$
31.  $\sum_{j=0}^{\infty} \frac{5^j x^{5j}}{(5j)!}$
32.  $\sum_{n=0}^{\infty} \frac{(x+5)^n}{(n+5)!}$