Math1210 Midterm 2 Extra Review Key

1. Let $y=2x^3-sec(\pi x)+\sqrt{x}$. If x changes from 1 to 1.035, approximately how much does y change?

Answer: 0.2275

2. Find the indicated derivative of the given functions.

(a) $D_x(\tan(4x^2+5x-1)\cos^2(3x))$

$= (sec^2(4x^2+5x-1))(8x+5)\cos^2(3x)+\tan(4x^2+5x-1)2\cos(3x)(-\sin(3x))(3)$

(b) $\frac{d}{dx}\left(\frac{x^4-3x^2+1}{x^3-\sqrt[4]{x}}\right)^5$

$= 5\left(\frac{x^4-3x^2+1}{x^3-\sqrt[4]{x}}\right)^4\left(\frac{(x^3-\sqrt[4]{x})(4x^3-6x)-(x^4-3x^2+1)(3x^2-\frac{1}{4\sqrt[4]{x^3}})}{(x^3-\sqrt[4]{x})^2}\right)$

(c) $f'(1)$ if $f(x)=(2x-\frac{1}{x})^3(4x^3-2)^4$ Answer: 528

(d) $\frac{dy}{dx}$ given $2x^4y+y^3=2x^2-6x$ Answer: $\frac{dy}{dx}=\frac{4x-6-8x^3y}{2x^4+3y^2}$

(e) $f'''(x)$ for $f(x)=(3x-4)^{\frac{2}{5}}$ Answer: $f'''(x)=\frac{1296}{125}(3x-4)^{-13/5}$

3. A softball diamond has the shape of a square with sides 40 ft. long. If a player is running from second base to third base at a speed of 20 ft/sec, at what rate is her distance from home plate changing when she is 30 ft from third base? Answer: -12 ft/sec

4. For (i) $f(x)=\frac{x}{1+x^2}$ and separately for (ii) $f(x)=(x^2-3)^2$, answer the following questions.

(a) Fill in the sign line for $f'(x)$.

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          --                 +                 --
------------|----------------------|---------------------
           -1                      1
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(b) Find all local min and max **__point(s)__**.

max point: (1, ½) min point: (-1, -½)

(c) Fill in the sign line for $f''(x)$.

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      –            +             –            +
------------|---------------|--------------|-------
          -√3              0              √3
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(d) Find all x-values of inflection point(s).

inflection point: (0,0), $(-\sqrt{3}, \frac{-\sqrt{3}}{4})$, $(\sqrt{3}, \frac{\sqrt{3}}{4})$

(e) Identify all critical **__points__** on the closed interval [-2, 2].

on closed interval, critical points are: (-2, -2/5), (1, ½), (-1, -½), and (2, 2/5)

5. Show that the tangent lines to the curves $y^2 = 4x^3$ and $2x^2 + 3y^2 = 14$ at (1, 2) are perpendicular to each other.

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5) answer:

① tangent line to $y^2 = 4x^3$ at $(1,2)$

$$2y\frac{dy}{dx} = 12x^2 \qquad \text{at } (1,2)$$

$$\frac{dy}{dx} = \frac{6x^2}{y} \implies m = \frac{6(1^2)}{2} = 3$$

② tangent line to $2x^2 + 3y^2 = 14$ at $(1,2)$

$$4x + 6y\frac{dy}{dx} = 0$$

$$6y\frac{dy}{dx} = -4x \qquad \text{at } (1,2)$$

$$\frac{dy}{dx} = \frac{-2x}{3y} \implies m = \frac{-2(1)}{3(2)} = \frac{-1}{3}$$

Since slope of 3 is negative reciprocal of slope of $\frac{-1}{3}$, then the two tangent lines are perpendicular.

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6. A 13 foot ladder is leaning against a vertical wall. If the bottom of the ladder is moving away from the wall at a constant rate of 0.5 ft/sec, how fast is the top of the ladder sliding down the wall when the top of the ladder is 5 feet above the ground? Answer: -1.2 ft/sec

7. Find $f^{(4)}(x)$ for $f(x) = (3x-7)^{\frac{5}{3}}$ Answer: $f^{(4)}(x) = 40(3x-7)^{\frac{-7}{3}}$

8. Let $y = \frac{2}{x}$. If x changes from 1 to 1.05, approximately how much does y change?

Answer: -0.1

9. The area of an equilateral triangle is decreasing at a rate of 2 square centimeters per second. Find the rate at which the length of a side is changing when the area of the triangle is $100\sqrt{3}$ square centimeters. (Note: Area of equilateral triangle with side length x is $A = \frac{\sqrt{3}}{4}x^2$.)

Answer: $\frac{-2}{10\sqrt{3}}$ cm/sec