2.2 Describing Sets/2.3 Other Set Operations and Their Properties

Set-- a group of things; a collection of objects
"elements"

element-- member of the set

sets are usually denoted by capital letters

list vs. set-builder notation

list

ex $\{1,2,3,4\}=A$

set-builder notation

$\{x \mid x=1,2,3 \text{ or } 4\}$

↑ "such that"

cardinality of a set--n(S) = the # of elements in set S.

ex $n(A)=4$

$B=\{\text{cat, dog, bird}\}$ $n(B)=3$

Symbols to know:

∈ element of ex cat ∈ B

∅ null set (a.k.a. empty set)

$\emptyset=\{\ \}$ $n(\emptyset)=0$

finite vs. infinite sets

$\mathbb{N}=$

ex $\{1,2,3,4,\ldots\}$

$n(\mathbb{N})=\infty$

equal sets: A = B

every element in set A is the same as the element in ~~set~~ B

$A = \{1, 2, 5\}$

$B = \{3, 4, 7\}$

$A \sim B$

$C = \{1, 2, 5\}$

$D = \{2, 1, 5\}$

$C = D$

equivalent sets: A ~ B (there's a one-to-one correspondence)

$(n(A) = n(B))$
for finite sets

subset: A ⊆ B "A is a subset of B"

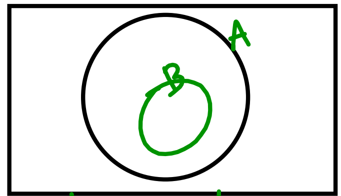
Ex.

(i.e. A is either a proper subset of B or equal to B)

A = {Joe, Megan, Yasmin}

B = {Joe, megan} C = {Megan, Yasmin, Jack}

Venn Diagram



B ⊆ A and B ⊂ A

C ⊄ A

3 < 5 ✓
3 ≤ 5 ✓

proper subset: A ⊂ B "A is a proper subset of B"

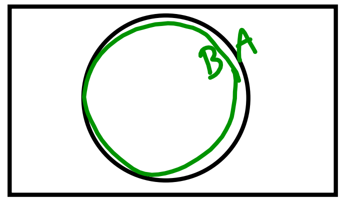
Ex.

every element in A is also in B

A = {1, 3, 5, 9, 10} A ⊂ B

B = {1, 3, 4, 5, 6, 7, 8, 9, 10, 21, 35}

Venn Diagram



B ⊆ A

set complement: $\overline{A}$ (A^c)

Ex. (not A)

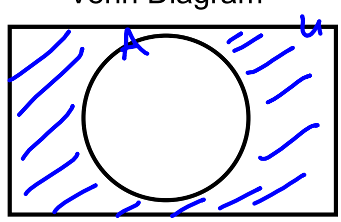
A = {Lara, Sean, Lauren, Galiba, Heather}

U = set of students in our 4010 class

$\overline{A}$ = {Baylee, Xavier, Clarissa, . . . , Katie}

U = Universal set

Venn Diagram



set difference: A - B (Note: your book calls this relative complement.)

Ex.

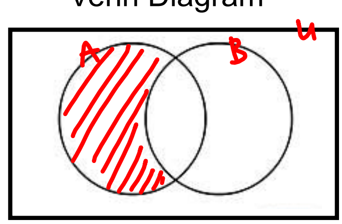
(gives all elements in A that are not in B)

A = {1, 3, 5, 7, 9, 11, 13}

B = {1, 2, 3, 4, 5, 6}

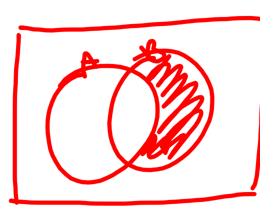
A - B = {7, 9, 11, 13}

Venn Diagram



A - B

B - A = {2, 4, 6}



B - A

set intersection: $A \cap B$ (and)

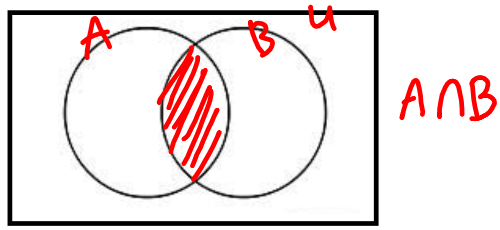
Ex.

$A = \{1, 2, 3, 4, 5, 6\}$

$B = \{3, 5, 7, 9, 11\}$

$A \cap B = \{3, 5\}$

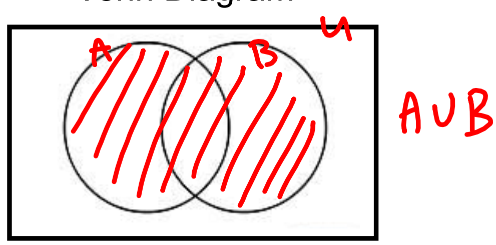
Venn Diagram



set union: $A \cup B$ ("or")

Ex. $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 9, 11\}$

Venn Diagram



disjoint sets: $A \cap B = \varnothing$

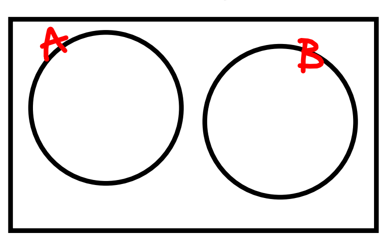
Ex. (sets that have nothing in common)

$A = \{1, 3, 5\}$

$B = \{2, 4, 6\}$

$A \cap B = \varnothing$

Venn Diagram



Commutativity: (order doesn't matter)

$A \cup B = B \cup A$ and $A \cap B = B \cap A$

Associativity: (we can regroup; regrouping doesn't matter)

$A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

Distributivity:

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

↑ ↑ ↑

calculate the number of subsets of a set A

n(A)	# subsets of A	# proper subsets
1	$2 = 2^1$	$1 = 2-1$
2	$4 = 2^2$	$3 = 4-1$
3	$8 = 2^3$	$7 = 8-1$
4	$16 = 2^4$	$16-1 = 15$
⋮		⋮
n	2^n	$2^n - 1$

ex $A = \{1, 2, 3\}$

$\emptyset$

$\{1\}, \{2\}, \{3\}$

$\{1,2\}, \{2,3\}, \{1,3\}$

$\{1,2,3\}$

ex $A = \{Levy\}$

subsets:

$\emptyset, \{Levy\}$

ex $A = \{L, m\}$

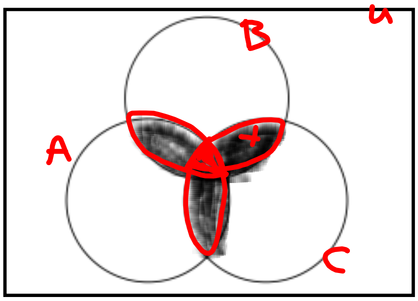
subsets:

$\emptyset, \{L\}, \{m\},$

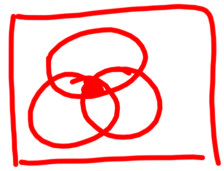
$\{L, m\}$

Ex: Given Venn Diagram, give set combination that would produce the shaded region.

(a)



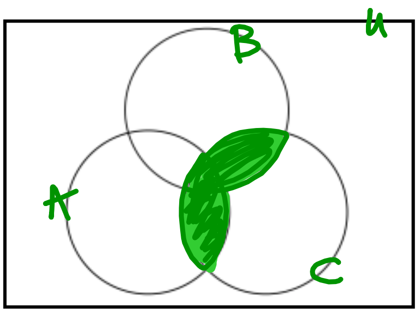
$A \cap B \cap C$



$(A \cap B) \cup (A \cap C) \cup (B \cap C)$

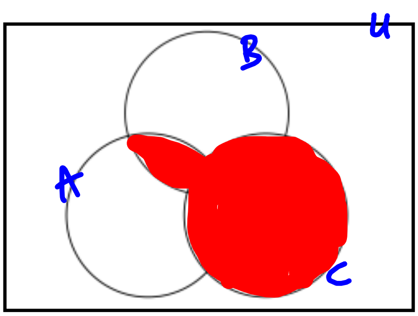
$A \cap B$ $A \cap C$ $B \cap C$

(b)



$(A \cap C) \cup (C \cap B)$

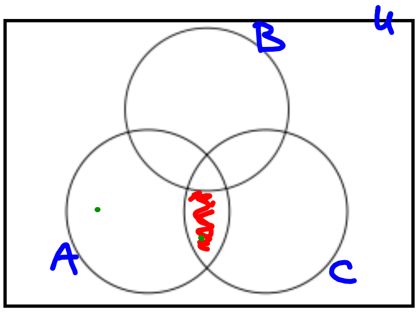
(c)

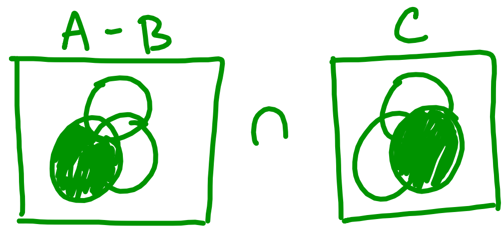


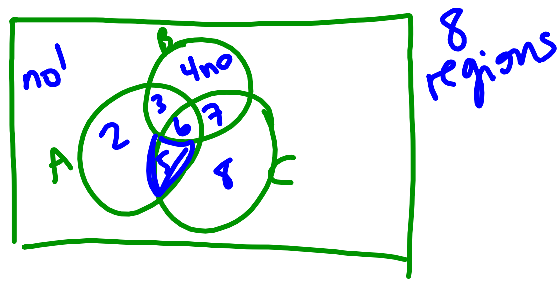
C or overlap of A and B

$C \cup (A \cap B)$

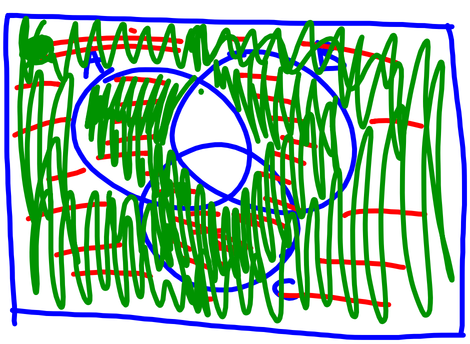
Ex: Shade in the region given by $(A - B) \cap C$

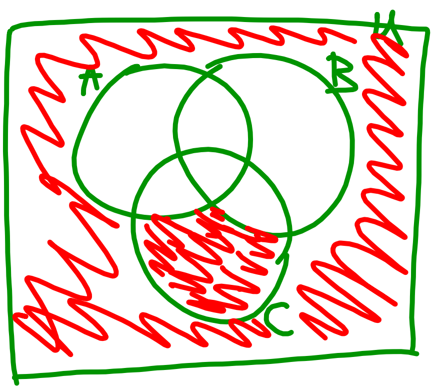






Ex Shade in $\bar{A} \cup \bar{B}$





$\bar{A} \cap \bar{B}$

(From homework)

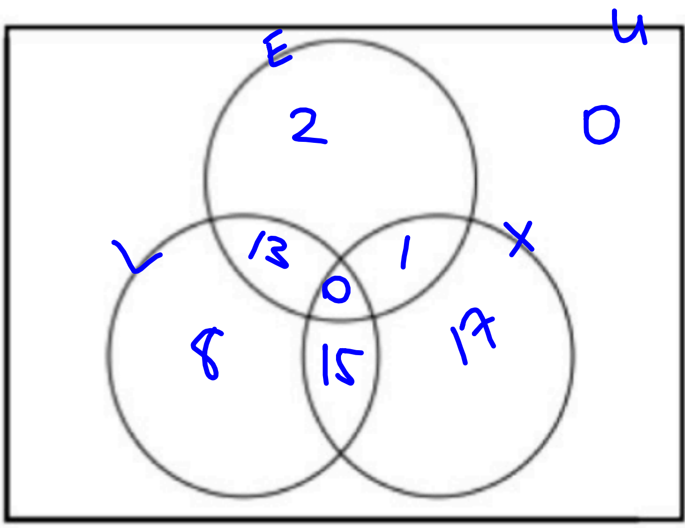
Use a Venn Diagram to summarize and analyze the data in each problem. Then use it to answer the questions.

1. Toward the middle of the season, peaches for canning tend to come in three types: early, late and extra late, depending on the expected date of ripening. During one week, the following data were recorded at a small peach receiving station.

- 16 trucks were dispatched carrying early peaches
- 36 trucks had late peaches
- 33 trucks had extra late peaches
- 13 trucks had early and late peaches
- 15 trucks had late and extra late peaches
- 1 truck had early and extra late peaches
- no trucks had all three types

✓ 16 E
✓ 36 L
✓ 33 X
✓ 13 E∩L
✓ 15 L∩X
✓ 1 E∩X
✓ 0 E∩L∩X

E = early
L = late
X = extra late



Determine the number of trucks:

(a) carrying only late peaches 8

(b) carrying only one variety of peaches 8+2+17=27

(c) carrying exactly two varieties of peaches 13+1+15=29

(d) Determine the total number of trucks. 56 (all #s from VD added)

2.2

A11) $A = \{a, b, c, d, e\}$

(a) # subsets $= 2^5 = 32$

(b) $32 - 1 = 31$ (# proper subsets)

(c) # subsets w/ a & e in them

$\{a, e, \_\_\_\}$ $2^3 = 8$

A7) (a) $\{101, 102, 103, \ldots, 1100\} = A$

$n(A) = 1100 - 100 = 1000$

(b) $\{1, 3, 5, \ldots, 1001\} = B$, $n(B) = \frac{1001-1}{2} + 1$

$= \frac{1002}{2} = 501$

1, 3, 5, 7, 9, 11

of #s $= 6 = \frac{12}{2}$

(c) $\{1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024\} = C$

$2^0, 2^1, 2^2, \ldots \ldots 2^{10}$

$n(C) = 11$

(d) $\{x \mid x = k^2, k = 1, 2, 3, \ldots 100\}$

$= \{1, 4, 9, 16, \ldots, 100^2\} = D$ $n(D) = 100$

(e) $\{i + j \mid i \in \{1, 2, 3\}$ and $j \in \{1, 2, 3\}\} = E$

$E = \{2, 3, 4, 5, 6\}$

$n(E) = 5$

$1+1=2$
$1+2=3$
$1+3=4$
$2+2=4$
$2+3=5$
$3+3=6$

2.2

MC8)

$A = \{1,2,3\}$ $B = \{4,5,6\}$ $U = \{1,2,3,4,5,6,7,8,9,10\}$

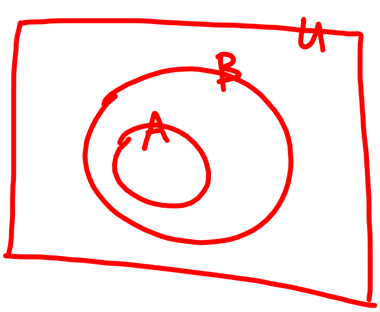
$\bar{A} = \{4,5,6,7,8,9,10\}$

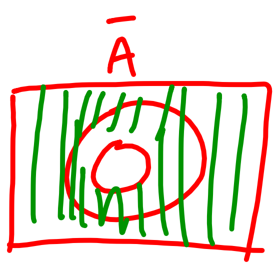
$\bar{B} = \{1,2,3,7,8,9,10\}$

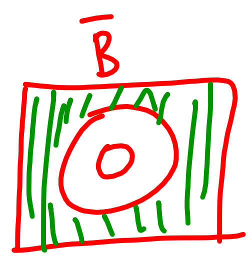
(a) $A = \{1,2,3\}$ $B = \{1,2,3,4,5\}$ $U = \{1,2,3,4,5,6,7\}$

$A \subset B$ $\bar{A} = \{4,5,6,7\}$ $\bar{B} = \{6,7\}$

(b) $\bar{B} \subset \bar{A}$







$\bar{B} \subset \bar{A}$

2.2

B6) $A = \{a, b, c\}$ $C = \{c, a, b\}$ $A = C$

$D = \{x \mid 1 \le x \le 3, x \in \mathbb{N}\} = \{1, 2, 3\}$

$I = \{x \mid x = 2n, n \in \mathbb{W}\} = \{0, 2, 4, 6, 8, \dots\}$

$L = \{x \mid x = 2n-1, n \in \mathbb{N}\} = \{1, 3, 5, 7, \dots\}$

A14) $\in$ $\notin$

(a) $0 \notin \emptyset$

(b) $\{1\} \notin \{1, 2\}$

$\{1\} \subset \{1, 2\}$

(c) $1024 \in \{x \mid x = 2^n, n \in \mathbb{N}\}$

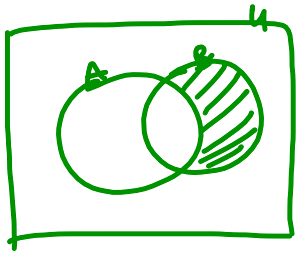
(d) $3002 \in \{x \mid x = 3n-1, n \in \mathbb{N}\}$
$= \{2, 5, 8, 11, \dots\}$

$3002 \overset{?}{=} 3n - 1$
$3003 = 3n$
$1001 = n$

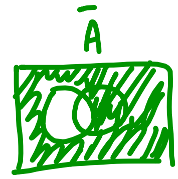
try $n = 1001$
$x = 3(1001) - 1$
$= 3003 - 1 = 3002$

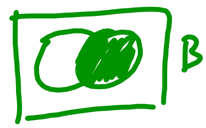
2.3A #9)

(a)

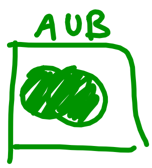


① ✓ $\bar{A} \cap B$

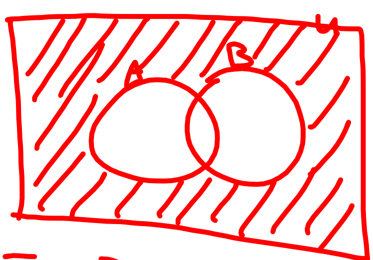




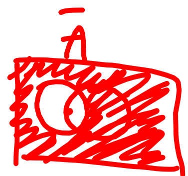
② $(A \cup B) - A$

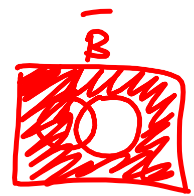


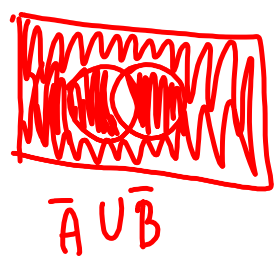
(b)



~~$\bar{A} \cup \bar{B}$~~

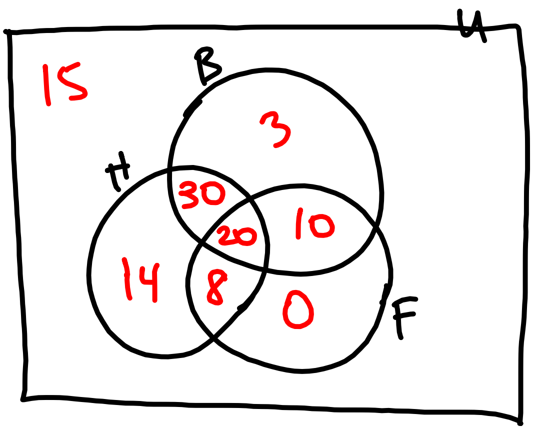






✓ $\bar{A} \cap \bar{B}$

WS #3)



15
3
30
20
10
14
———
92

- total 100
- ✓ 63 B
- ✓ 62 not F
- ✓ 18 not F and not H
- ✓ 30 F and B
- ✓ 28 not H
- ✓ 20 all three
- ✓ 15 not any

(a) 14

(b) 20+8=28

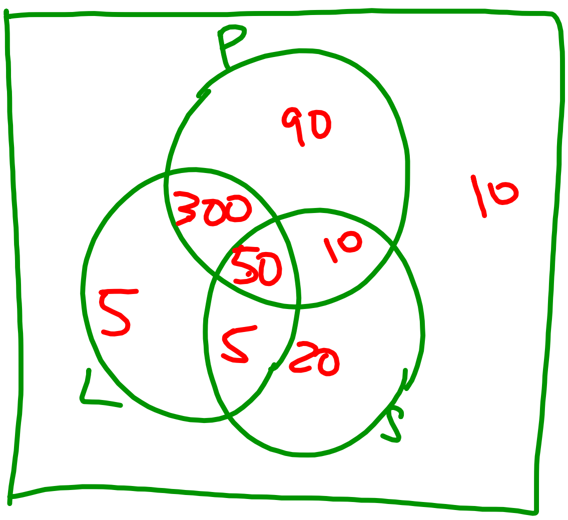
(c) 30+10+8+20=68

WS #6

P=passed

L= liked

S=signed up



✓· 450 P

✓· 10 not P and L

✓· 25 not P and S

✓· 55 L and S

✓· 60 P and S

✓· 350 P and L

✓· 300 P and L and not S

✓· 130 not L

(a) 90+10+20+50+5+300+5+10=490

(b) 360

(c) 5+5+20+10=40

(d) not P and not L and not S 10

(e) not L and P 90+10=100

(f) P and L and S 50

2.3 HW

A1) $N = \{1, 2, 3, 4, \ldots\}$ $A = \{1, 3, 5, 7, \ldots\}$

$B = \{2, 4, 6, 8, \ldots\}$

(a) $A \cup C = A = C$ $C = \{1, 3, 5, 7, \ldots\}$

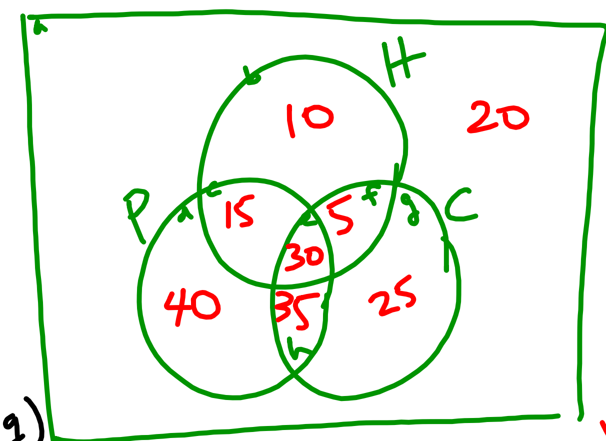
(b) $A \cup B = \{1, 2, 3, 4, 5, \ldots\} = N$ (c) $A \cap B = \phi$

A3) (c) $\overline{A \cap B} = \bar{A} \cap \bar{B}$ false

MC#14) "A+B is same as A∪B"

not because there is no such thing as addition of sets.

WS #5)



- 180 total
- ✓ 60 H
- ✓ 95 C
- ✓ 120 P
- ✓ 55 P and not C
- ✓ 45 H and P
- ✓ 10 H only
- ✓ 30 all three

(a) 20

(b) 25 (section g)

(c) 160 = 180 − 20

(d) 30 + 5 = 35

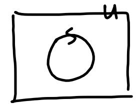
(e) 5

(f) 10 + 40 + 25 = 75

(g) 180 − 10 − 20 = 150

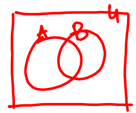
(h) 15 + 5 + 35 = 55

2.3 A#6)
(a) $S \cup \bar{S} = U$
(b) $\bar{U} = \emptyset$
(c) $S \cap \bar{S} = \emptyset$
(d) $\emptyset \cap S = \emptyset$

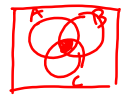


B2)
(a) $A - B = B - A$? no

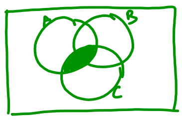
(b) $A - B = A \cap B$? no



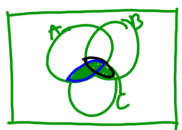
(c) $A \cap (B \cap C) = (A \cap B) \cap C$? yes



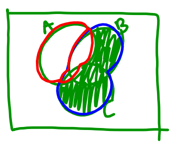
(d) $B \cup \emptyset = B \cap B$? yes
$B = B$ ✓

B9)
(a)


$A \cap C$

(b)


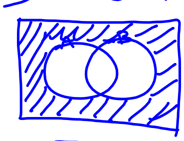
$(A \cap C) \cup (B \cap C)$

(c)


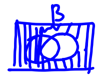
$(B \cup C) - A$

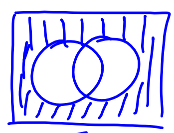
$(B \cup C) \cap \bar{A}$

B14) $\overline{A \cup B} = \bar{A} \cap \bar{B}$



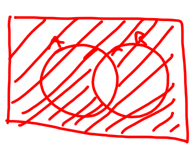
$\overline{A \cup B}$



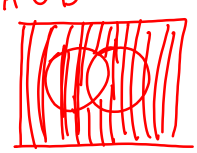


$\bar{A} \cap \bar{B}$

$\overline{A \cap B} = \bar{A} \cup \bar{B}$



$\bar{A} \cup \bar{B}$



$U = \{1, 2, 3, \ldots, 9\}$

(c) $A = \{1, 2, 3, 4, 5, 6\}$ $B = \{3, 4, 5, 7, 8\}$

$\overline{A \cup B} = \{9\}$ $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$\overline{A \cap B} = \{1, 2, 6, 7, 8, 9\}$ $A \cap B = \{3, 4, 5\}$ $\bar{B} = \{1, 2, 6, 9\}$

$\bar{A} = \{7, 8, 9\}$

$\bar{A} \cap \bar{B} = \{9\}$ $\bar{A} \cup \bar{B} = \{1, 2, 6, 7, 8, 9\}$

2.2/2.3 Quiz 4

1) (a) 5 (b) $2^5 = 32$

(b) 200 (b) 2^{200}

2) (a) $\phi \subset A$ (A non-empty) T

(b) $\phi = \{\phi\}$ F

$\phi = \{\ \} \neq \{\phi\}$

(c) $\{2\} \in \{2,3,4\}$ F

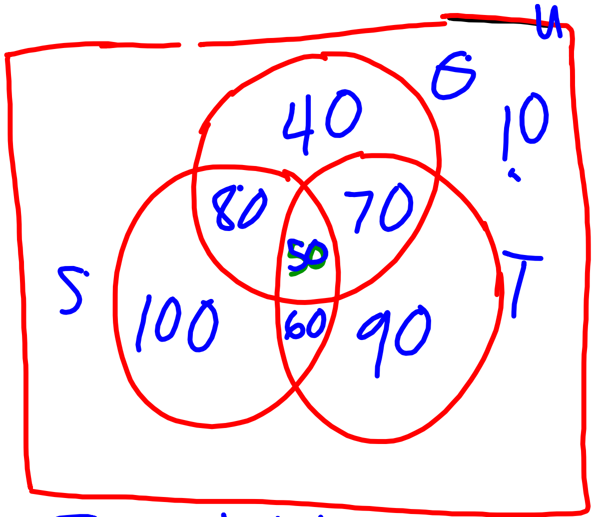
fixed ① $2 \in \{2,3,4\}$ or ② $\{2\} \subset \{2,3,4\}$

(d) $\{x, y, z\}$ equivalent to $\{1,2,3\}$ T

(e) $\overline{A \cup B} = \bar{A} \cup \bar{B}$ F

fixed ① $\overline{A \cup B} = \bar{A} \cap \bar{B}$ or ② $\bar{A} \cup \bar{B} = \overline{A \cap B}$

4)



- 240 G
- 290 S
- 270 T
- 80 G AND S only
- 70 G AND T only
- 60 S AND T only
- 50 all 3

500 total srs w/CC