6.1 Rational Numbers

set of rational numbers = **Q** = $\left\{ \frac{a}{b} \;\middle|\; a, b \in \mathbb{Z},\ b \neq 0 \right\}$ such that

$\mathbb{N} \rightarrow \mathbb{W} \rightarrow \mathbb{Z} \rightarrow \mathbb{Q}$

Vocabulary-- $\frac{a}{b}$

numerator (a) "top in $\frac{a}{b}$ form"

denominator (b) "bottom in $\frac{a}{b}$ form"

proper fraction ex $\frac{1}{2}$

numerator < denominator

improper fraction ex $\frac{3}{2}$

numerator $\geq$ denominator

ex $\frac{3}{4}$

"3 divided by 4"

or "4 divided into 3"

We use fractions in two ways:

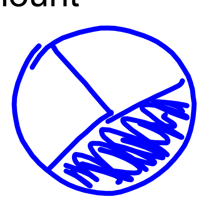
1. part-to-whole

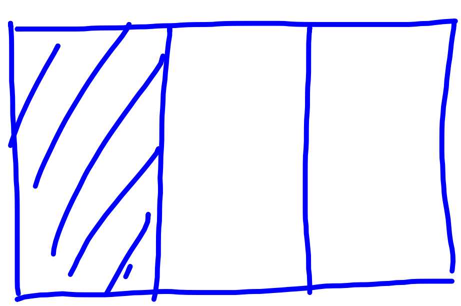
We need to consider: (a) the whole, (b) the number of equal-sized parts that the whole has been divided into, and (c) the number of parts we have.

ex I ate 3 out of 5 pieces (equal sized).

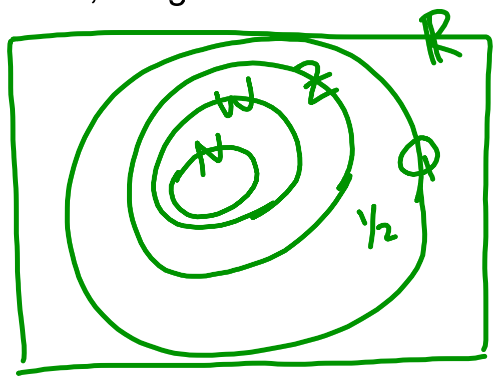
2. relative amount

ex $\frac{1}{3}$

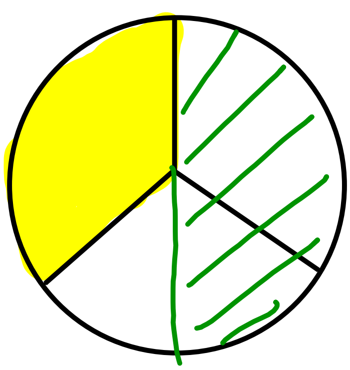


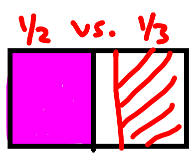


Draw a Venn Diagram to display the relationship between the natural numbers, whole numbers, integers and rational numbers.



Max claims that $\frac{1}{3} > \frac{1}{2}$ because in the below figure, the shaded portion for $\frac{1}{3}$ is larger than the shaded portion depicting $\frac{1}{2}$. Is he correct? It not, how would you help him?





Equivalent fractions==> fractions that represent the same relative amount

$$\frac{a}{b}=\frac{an}{bn} \text{ for any nonzero n}$$

mult. identity

ex $\frac{3}{5}\left(\frac{2}{2}\right)=\frac{6}{10}$

How to decide if fractions are equal:

$$\frac{a}{b}=\frac{c}{d} \text{ iff } ad=bc \text{ (assuming } b\neq 0 \text{ and } d\neq 0)$$

Other ideas?

ex $\left(\frac{3}{3}\right)\frac{3}{6} \overset{?}{=} \frac{9}{18}$ (mult. by 1 to get common denominator)

Ex 1. Are these true or false statements? Why?

(a) $\frac{16}{56}=\frac{2}{7}$ true $\quad \frac{\cancel{16}^{2}}{\cancel{56}_{7}}=\frac{2}{7}$

(b) $\frac{2}{6}\neq\frac{1}{4}$ $\quad \frac{2}{6}=\frac{1}{3}$

Ex 2. Create three other equivalent fractions for $\frac{4}{9}$.

$\frac{8}{18}, \frac{12}{27}, \frac{20}{45}$

Ordering fractions:

1. $\dfrac{a}{c} < \dfrac{b}{c}$ iff $a < b$

2. $\dfrac{a}{b} > \dfrac{c}{d}$ iff $ad > bc$ (assuming b, d > 0)

3. If $\dfrac{a}{b} < \dfrac{c}{d}$, then $\dfrac{a}{b} < \dfrac{a+c}{b+d} < \dfrac{c}{d}$ (assuming that b, d > 0).

ex $\dfrac{3}{7} < \dfrac{9}{10} \Rightarrow \dfrac{3}{7} < \dfrac{12}{17} < \dfrac{9}{10}$

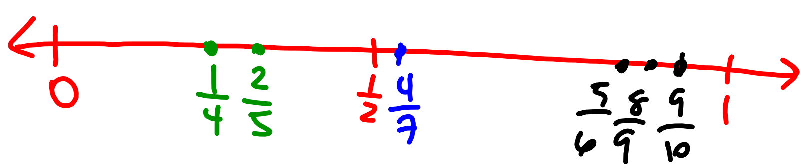
$3(170) = 510$
$12(70) = 840$

$9(7)(17) = 1071$

$\dfrac{3(170)}{7(170)} < \dfrac{12(70)}{17(70)} < \dfrac{9}{10}\left(\dfrac{7(17)}{7(17)}\right)$

Ex 3. Order these rational numbers from least to greatest and plot them on a number line.

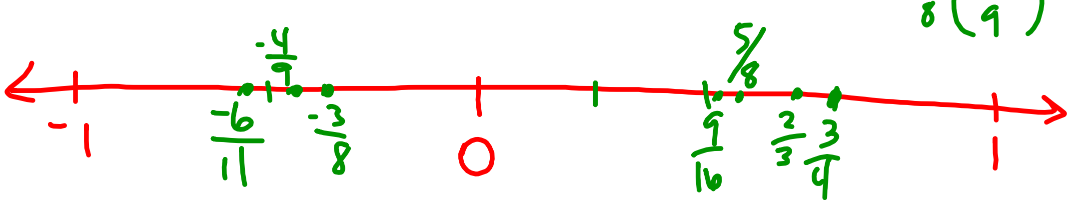
(a) $\dfrac{4}{7}, \dfrac{9}{10}, \dfrac{8}{9}, \dfrac{1}{4}, \dfrac{2}{5}, \dfrac{5}{6}$



(b) $\dfrac{3}{4}, \dfrac{9}{16}, \dfrac{5}{8}, \dfrac{2}{3}, -\dfrac{3}{8}, -\dfrac{6}{11}, -\dfrac{4}{9}$

$-\dfrac{4}{9}\left(\dfrac{8}{8}\right) = \dfrac{-32}{72}$

$-\dfrac{3}{8}\left(\dfrac{9}{9}\right) = \dfrac{-27}{72}$



Ex 4. (a) Is this true or false and why? $\frac{7}{8} < \frac{10}{11}$

true

$\Leftarrow \frac{77}{88} < \frac{80}{88}$

(b) Tell whether each of these fractions is closer to 0, one-half or 1.

$\frac{3}{8}, \frac{2}{7}, \frac{1}{3}, \frac{21}{50}, \frac{4}{5}, \frac{7}{11}, \frac{31}{181}, \frac{3}{4}$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ 0 $\frac{1}{2}$ and 1

(c) Fill in the blank with < , > or =. $\left(\frac{9}{9}\right)\frac{7}{8}$ __>__ $\frac{5}{9}\left(\frac{8}{8}\right)$

$\frac{63}{72}$ $\frac{40}{72}$

Simplifying Fractions

A rational number, a/b, is in simplest form iff the GCF(a,b) = 1, assuming b is nonzero.

Ex 5. Simplify these fractions.

(a) $\frac{12}{18} = \frac{2}{3}$

(b) $\frac{42}{52} = \frac{21}{26}$

(c) $\frac{294}{63} = \cancel{\frac{32}{7}}$ $\frac{14}{3}$ or $4\frac{2}{3}$

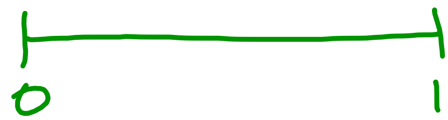
$\frac{98}{21}$

(d) $\frac{2^2 3^4 5^3}{2^3 3 \cdot 5^2} = \frac{3^3 \cdot 5}{2} = \frac{135}{2}$ or $67\frac{1}{2}$

(e) $\frac{\cancel{14}^{7} a b^2}{\cancel{20}_{10} a^5 b^3} = \frac{7}{10a^4 b}$

(f) $\frac{8+x^2}{2x}$ it is already simplified

Explain why there are infinitely many rational numbers between any two rational numbers.



$\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots$

$\frac{2}{3}, \cancel{\frac{2}{4}}, \frac{2}{5}, \cancel{\frac{2}{6}}, \frac{2}{7}, \frac{2}{9}, \frac{2}{11}, \frac{2}{13}, \ldots$

6.1A 18) (b) $-\frac{1}{5}$, $-\frac{19}{36}$, $\frac{-17}{30}$

$\text{lcm}(5, 36, 30)$

$= 5 \cdot 6 \cdot 6$

5	5	36	30
6	1	36	6
6	1	6	1
	1	1	1

$\frac{-1}{5} = \frac{-36}{180}$ (biggest) $\qquad \frac{-19}{36} \qquad \frac{-17}{30}$ (smallest)

$\frac{-1}{5}\left(\frac{36}{36}\right) = \frac{-36}{5(36)}$

$\frac{-17}{30}\left(\frac{6}{6}\right) = \frac{-102}{180}$

$\frac{-19}{36}\left(\frac{5}{5}\right) = \frac{-95}{180}$

6.1A 11b) $\frac{\cancel{18}^{\,2}}{\cancel{54}_{\,6}}$ ___ $\frac{23}{69}$

$\frac{1}{3}$ $\underline{=}$ $\frac{1}{3}$

ex $\frac{7}{9} \underline{<} \frac{4}{5}$

$\frac{7}{9}\left(\frac{5}{5}\right)$, $\frac{4}{5}\left(\frac{9}{9}\right)$

$= \frac{35}{45}$, $= \frac{36}{45}$

$\frac{7}{9}$ ___ $\frac{4}{5}$ $\quad 7 \cdot 5 \quad 9 \cdot 4$

$35 \quad 36$

6.1 MC#3)

$\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$

$$\frac{1+2+3}{2+4+6} = \frac{6}{12} = \frac{1}{2}$$

$\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$ $\quad \frac{1+2+3+4}{2+4+6+8} = \frac{10}{20} = \frac{1}{2}$

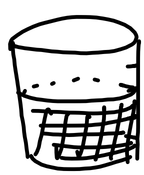
$\frac{n}{2n}$ $\quad n \in \mathbb{Z}$, p, m $\quad n \neq 0, m \neq 0, p \neq 0$, etc.

$\frac{n}{2n}, \frac{m}{2m}, \frac{p}{2p}, \ldots$ $\quad \frac{n+m+p+\cdots}{2n+2m+2p+\cdots} = \frac{\cancel{n+m+p+\cdots}}{2(\cancel{n+m+p+\cdots})} = \frac{1}{2}$

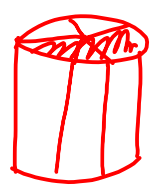
MC#1) "because it's a ratio $\frac{3}{4} : 1$"

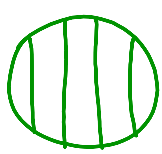
$\frac{3}{4}(4) = 3$ $\quad$ repeatedly added $\frac{3}{4}$, 4 times

B3f)

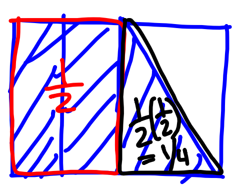


illustrate $\frac{3}{5}$





6.1A 4b)



represents $\frac{3}{4}$?