

Name Key

Instructions: Please show all of your work as partial credit will be given where appropriate, and there may be no credit given for problems where there is no work given. Make sure all answers are simplified! **No calculators allowed.** (Total of 240 points.)

1. (30 points) Answer each true or false question by circling the correct answer. For every answer, justify your reasoning on the given blank line!!!

(a) $-\left(\frac{8}{5}\right)^{-1}$ is bigger than $\frac{-4}{9}$. $\frac{45}{72} = \frac{5(9)}{8(9)} > \frac{4(8)}{9(8)} = \frac{32}{72}$ T or **F** (circle one)
 $= -\frac{5}{8} \Rightarrow -\frac{5}{8} < -\frac{4}{9}$

(b) $-(-(-x))$ is a negative number. T or **F** (circle one)
 if $x = -3$, we'd have $-(-(-(-3))) = 3$

(c) $\left(\frac{16}{25}\right)^{-1/2}$ is closer to 1 than $\frac{1}{2}$. **T** or F (circle one)
 $= \left(\frac{25}{16}\right)^{1/2} = \frac{5}{4}$

(d) The decimal 0.0061 is read "61 thousandths." T or **F** (circle one)
 $\frac{61}{10000}$ it's 61 ten-thousandths

(e) The set $\{f, u, n\}$ has 9 total subsets. T or **F** (circle one)
 it has $2^3 = 8$ total subsets

(f) If $a, b,$ and c are real numbers, and $a \leq b$, then $ac \leq bc$. T or **F** (circle one)
 only true if $c > 0$.

(g) $(a-b)^2 = a^2 + b^2$ T or **F** (circle one)
 $(a-b)^2 = a^2 - 2ab + b^2$

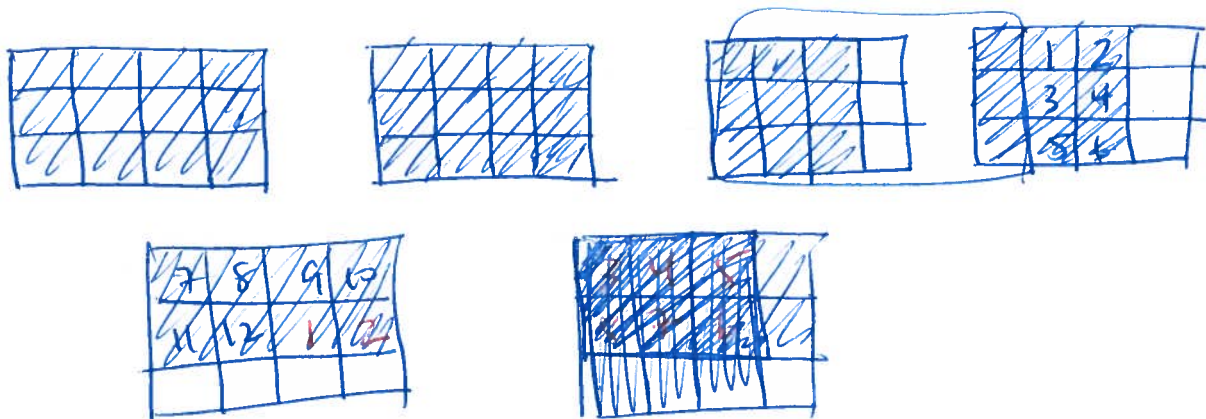
(h) The square root of any prime number is irrational. **T** or F (circle one)

2. For all parts of this problem, I will dictate which method to use. (For the rest of the test, it will be your choice of method.) If you can't remember how to use the specified method, use any method for 75% of the credit for these problems.

(a) (10 points) Compute $2\frac{2}{3} \times 1\frac{3}{4}$ using an area model.

There are a couple ways you can do this. Here's one.

$$2\frac{2}{3} \cdot 1\frac{3}{4} = (2 + \frac{2}{3})(1 + \frac{3}{4}) = 2(1) + 2(\frac{3}{4}) + \frac{2}{3}(1) + \frac{2}{3}(\frac{3}{4})$$



$$= 2 + 1 + 1 + \frac{8}{12} = 4\frac{2}{3}$$

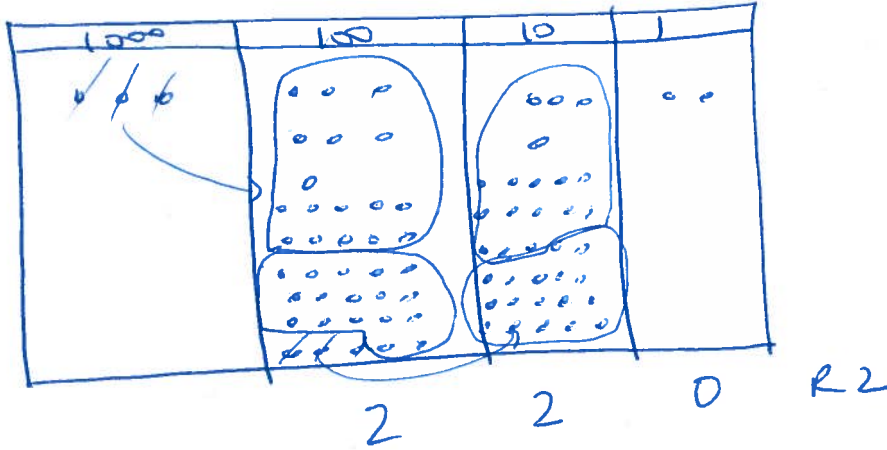
(b) (10 points) Compute $2421_5 - 1433_5$ using the standard algorithm.

$$\begin{array}{r} ^{\text{13}} ^{\text{11}} \\ 2421_5 \\ - 1433_5 \\ \hline 433_5 \end{array}$$

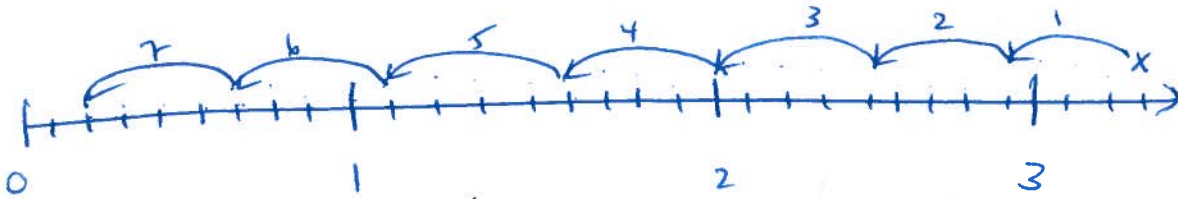
(Note: This is #2 continued.)

(c) (10 points) Compute $3742 \div 17$ using a chip abacus.

$$3742 \div 17 = 220 \text{ R}2$$



(d) (10 points) Compute $3\frac{1}{3} \div \frac{4}{9}$ using a number line.



$$3\frac{1}{3} \div \frac{4}{9} = 7\frac{1}{2}$$

(since there is half
of $\frac{4}{9}$ left)

3. Simplify the following expressions.

(a) (10 points) $\frac{(3^{-4})^4 (81^{\frac{3}{2}}) (9^{-2})}{(27^2) \cdot \frac{1}{3^5}}$ (Give answer as a base to a single exponent.)

$$= \frac{3^{-16} (3^4)^{3/2} (3^2)^{-2}}{(3^3)^2 \cdot 3^{-5}} = \frac{3^{-16} \cancel{3^6} 3^{-4}}{\cancel{3^6} 3^{-5}}$$

$$= \frac{3^5}{3^{16} 3^4} = \frac{3^5}{3^{20}} = \frac{1}{3^{15}} \text{ or } 3^{-15}$$

Answer 3(a): 3⁻¹⁵

(b) (10 points) $-1\frac{2}{3} \cdot \left(\frac{1}{4} - \frac{7}{8}\right) \div 2\frac{1}{6} - 5 + 1$

$$= -\frac{5}{3} \left(\frac{2}{8} - \frac{7}{8}\right) \div \frac{13}{6} - 5 + 1$$

$$= -\frac{5}{3} \left(\frac{-5}{8}\right) \cdot \frac{6}{13} \cdot \frac{1}{8} + 1$$

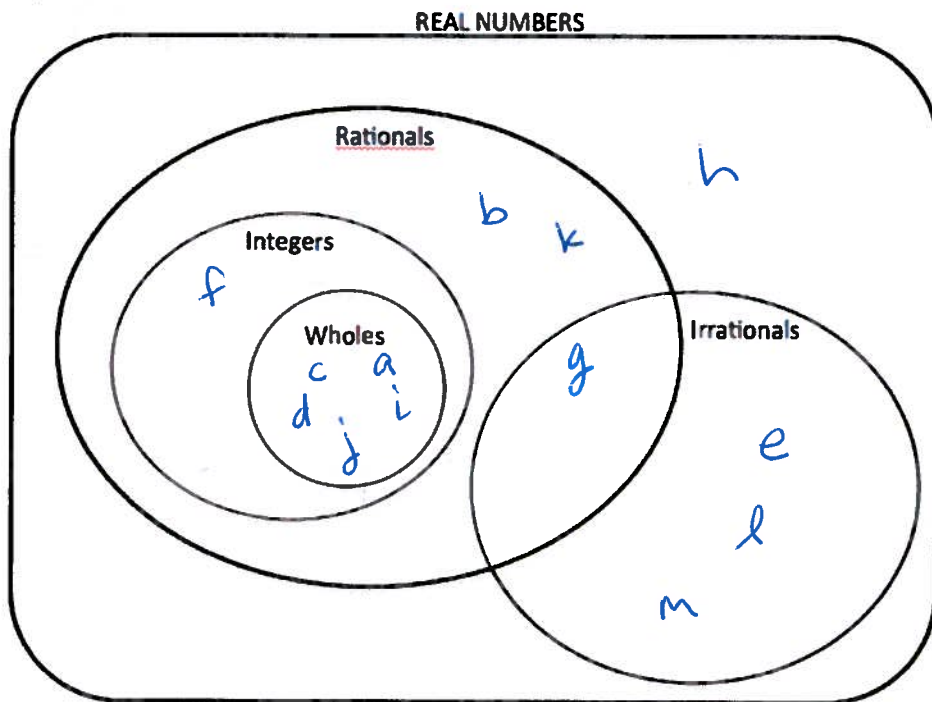
$$= \frac{-5}{52} + 1 = \frac{-5}{52} + \frac{52}{52} = \frac{47}{52}$$

Answer 3(b): $\frac{47}{52}$

4. (15 points)

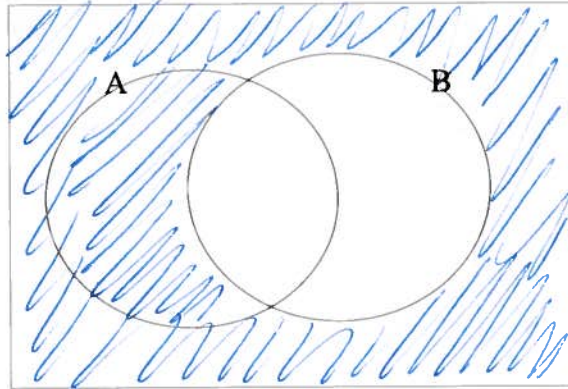
Place each of the following numbers in the appropriate location of the Venn Diagram provided for you. Please just write in the corresponding letter in the Venn Diagram (instead of the actual number).

- (a) $\sqrt{49} = 7$
- (b) $-3.415656565656\dots = -3.\overline{4156}$
- (c) The additive identity = 0
- (d) The multiplicative identity = 1
- (e) $4.23223222322223\dots$
- (f) -4
- (g) No numbers go in this space.
- (h) No numbers go in this space either.
- (i) $0.\overline{9} = 1$
- (j) $0 \div 53 = 0$
- (k) $\frac{2}{7}$
- (l) $\frac{3}{\pi}$
- (m) $\sqrt{48}$





5. (10 points) Shade in $\overline{A \cap B} - B$.



6. (15 points) State whether the numbers below are rational or irrational. If the number is rational, write it in its simplest **fraction** form (not as a decimal, but either improper fraction or mixed number is fine).

(a) -3.0025

$$= -3 \frac{25}{10000} = -3 \frac{1}{400} \quad (\text{or } -\frac{1201}{400})$$

Rational or Irrational: rational

Simplified form (if rational): $-3 \frac{1}{400}$

(b) -19.01012012301234...

Rational or Irrational: irrational

Simplified form (if rational): N/A

(c) 35.4015151515...

$$n = 0.40\overline{15}$$

$$10000n = 4015.\overline{15}$$

$$-100n = 40.\overline{15}$$

$$9900n = 3975$$

Rational or Irrational: rational

Simplified form (if rational): $35 \frac{53}{132}$

$$n = \frac{3975}{9900} = \frac{159}{396} = \frac{53}{132}$$

7. (10 points) Do ONE of these problems (you choose).

Grade: Y or N (circle one)

(a) If N is some negative integer, will $N^4 + N^3$ be positive, negative, neither or is it impossible to determine? Justify your answer.

N^4 will be positive

N^3 will be negative

① if $N \leq -2$, then $|N^4| > |N^3| \Rightarrow N^4 + N^3 > 0$

but ② if $N = -1$, then $N^4 + N^3 = 0$

Grade: Y or N (circle one)

(b) If E is an even number and O is an odd number (neither are zero), then is

$O^2 - \frac{E}{O} + E + OE$ even, odd, neither or impossible to determine? Justify your answer.

O^2 is odd, E is even, OE is even

but $\frac{E}{O}$ is a fraction (possibly proper or improper) and it will ^{possibly} have a remainder

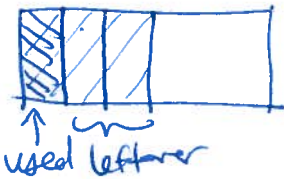
\Rightarrow impossible to determine.

8. (10 points) Does the following question have $\frac{1}{2} - \frac{1}{3}$ as a solution? If it does, explain why and find a difference using some sort of picture. If it does not, then give the solution, using a picture, and explain how the problem should be changed so that the solution is the given one.

Zelha pours $\frac{1}{2}$ cup of water into an empty bowl. Then she uses $\frac{1}{3}$ of the water to water her plants. How much water is left in the bowl?

NO, because " $\frac{1}{3}$ of the water" means $\frac{1}{2}(\frac{1}{3}) = \frac{1}{6}$ used to water her plants

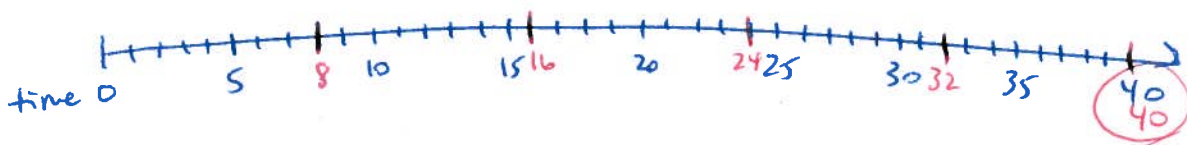
$$\Rightarrow \text{water left} = \frac{1}{2} - \frac{1}{6} = \frac{2}{6} = \frac{1}{3} \text{ cup}$$



To give proposed solution, change it to:

"Zelha pours $\frac{1}{2}$ cup of water into a bowl. Then she uses $\frac{1}{3}$ of a cup to water the plants. How much water is left?"

9. (10 points) You are camping underneath the stars on a warm summer night. You notice that a frog croaks every five seconds, while an owl hoots every eight seconds. Suppose that you hear the creatures make their noises at the exact same time. Then, after how many seconds will they once again be in sync?



$$\text{LCM}(5, 8) = 40$$

Answer: 40 sec

10. (10 points) You go to Costco and buy a large container of chocolate almonds, holding 36 cups of the candy. In order to maintain the health of your family, you decide to eat only $1\frac{3}{5}$ cups per day between you and your family members. How many days will the container of chocolate almonds last your family? And, how many cups (or what fraction of a cup) of chocolate almonds are leftover?

$$36 \div 1\frac{3}{5} = 36 \div \frac{8}{5} = 36 \cdot \frac{5}{8} = \frac{45}{2} = 22\frac{1}{2} \text{ servings or days}$$

$$\frac{1}{2} \text{ a day} = \frac{1}{2} \left(1\frac{3}{5}\right) = \frac{1}{2} \left(\frac{8}{5}\right) = \frac{4}{5} \text{ cups left}$$

days it lasts: 22

leftover: $\frac{4}{5}$ cups

11. (10 points) Saeed took 19 orders for either a burger (\$3.10) or a hot dog (\$1.80), but he forgot how many of each were ordered. If the bill comes to \$43.30, then how many of each were ordered?

$$b = \# \text{ burger} \quad h = \# \text{ hot dogs} \quad b+h=19$$

$$3.10b + 1.8h = 43.30$$

$$3.1(19-h) + 1.8h = 43.3$$

$$31(19-h) + 18h = 433$$

$$18h - 31h = 433 - 31(19)$$

$$-13h = 433 - 589$$

$$+13h = -156$$

$$h = 12$$

$$\Rightarrow b = 19 - 12 = 7$$

$$\begin{aligned} 31(19) &= 30(19) + 1(19) \\ &= 570 + 19 \\ &= 589 \end{aligned}$$

$$\begin{array}{r} 12 \\ 13 \overline{)156} \\ \underline{13} \\ 26 \\ \underline{26} \\ 0 \end{array}$$

Number of hot dogs: 12

Number of Hamburgers: 7

12. (10 points) Brielle paid \$11 for a shirt that was discounted by 45%. What was the original price of the shirt?

$$\underline{11} \text{ is } \frac{55}{\text{percent she paid}} \% \text{ of } \underline{x}$$

$$11 = 0.55x$$

$$\frac{11}{0.55} = x$$

$$\Rightarrow x = \frac{1100}{55} = \$20$$

Original price: \$20

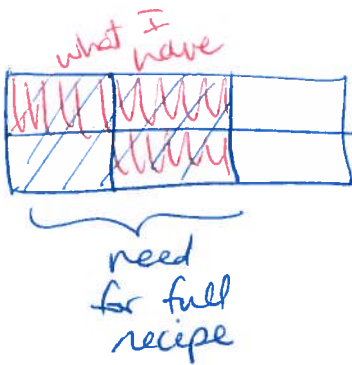
13. (10 points) Explain the relationship between rational numbers and decimals.

all rational #s can be written as either terminating decimals or repeating (nonterminating) decimals

14. (10 points) A muffin recipe calls for $\frac{2}{3}$ cup of sugar. You realize you only have $\frac{1}{2}$ cup of sugar left in you cupboard. If you are to use exactly the sugar you have, what fraction of the recipe can you make? The recipe also calls for $1\frac{1}{2}$ cups of flour. How much flour will you have to use for your muffins?

$\frac{1}{2}$ is what fraction of $\frac{2}{3}$?

$$\frac{1}{2} = \frac{2}{3}x \Rightarrow x = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$$



flour:

$$\frac{3}{4} \left(1\frac{1}{2}\right) = \frac{3}{4} \left(\frac{3}{2}\right) = \frac{9}{8} = 1\frac{1}{8}$$

Fraction of recipe you can make: $\frac{3}{4}$

How much flour you need: $1\frac{1}{8}$ cups

15. (10 points) When I ordered the gourmet cupcakes for my class, the total cost was \$54.21 for 30 cupcakes, including tax. When I called to confirm, I told him I actually only needed 24 cupcakes. Assuming the cost per cupcake remains the same, what should my total bill be?

$$\frac{54.21}{30} = \frac{x}{24} \Rightarrow x = \frac{24(54.21)}{30}$$

$$= \frac{216.84}{5}$$

$$\begin{array}{r} 43.368 \\ 5 \overline{)216.84} \\ \underline{20} \\ 16 \\ \underline{15} \\ 18 \\ \underline{15} \\ 34 \\ \underline{30} \\ 40 \end{array}$$

(must round up to nearest penny)

Total bill: \$43.37

A. Choose 2 out of the following 3 problems to do. Indicate (by circling "Y" or "N") clearly which problems you want graded. I will only grade 2 problems !!!

Each problem is worth 10 points.

A1. Grade: *Y* or *N* Use the problem $\frac{4}{7} \div \frac{2}{3} = ?$ to explain why the "invert and multiply" rule works.

$$\frac{4}{7} \div \frac{2}{3} = \frac{\frac{4}{7}}{\frac{2}{3}} = \frac{\frac{4}{7}}{\frac{2}{3}} \left(\frac{\frac{3}{2}}{\frac{3}{2}} \right) = \frac{\frac{4}{7} \cdot \frac{3}{2}}{1} = \frac{4}{7} \cdot \frac{3}{2}$$

multiply by 1

A2. Grade: *Y* or *N* Use a logical argument to explain why $\frac{5}{0}$ and $\frac{0}{0}$ are undefined, and describe the difference in the "flavors" of undefined for these two expressions.

① $\frac{5}{0} = ? \Leftrightarrow 5 = 0 \cdot ?$ but nothing works here because anything times zero is 0, so it's undefined.

② $\frac{0}{0} = ? \Leftrightarrow 0 = ? \cdot 0$ but any finite # would work here because anything times zero is zero so it's undefined

A3. Grade: Y or N Prove that $\sqrt{5}$ is irrational. (Write enough for each step that convinces me you really understand.)

pf Assume $\sqrt{5}$ is rational.

Then there exist $a, b \in \mathbb{Z}$, $b \neq 0$, such that $\gcd(a, b) = 1$ and
$$\sqrt{5} = \frac{a}{b} \quad (\text{by defn of rational } \#).$$

$$\Rightarrow 5 = \frac{a^2}{b^2} \Rightarrow 5b^2 = a^2$$

but a^2 has an even # of prime factors and b^2 also has an even # of prime factors $\Rightarrow 5b^2$ has an odd # of prime factors.

$\Rightarrow 5b^2 \neq a^2$ since the fundamental theorem of arithmetic guarantees only one unique prime factorization for any #.

\Rightarrow the contradiction means our original assumption is wrong $\Rightarrow \sqrt{5}$ is irrational.

