2-d

To find a tangent line to a curve, at $x=x_0$.

Explicit function

$y=f(x)$

full pt: $(x_0, f(x_0))$
i.e. $y_0=f(x_0)$

tangent line:

$$\boxed{y=y_0+f'(x_0)(x-x_0)}$$

$\Leftrightarrow y-y_0=f'(x_0)(x-x_0)$

$0=f'(x_0)(x-x_0)-(y-y_0)$

(★) $0=\langle f'(x_0),-1\rangle\cdot\langle x-x_0, y-y_0\rangle$

If we let (redefine this curve as implicit fn)

$F(x,y)=f(x)-y=0$, then

we get $F_x=f'(x)$
and $F_y=-1$

$\Rightarrow$ (★) becomes

$0=\langle F_x(x_0,y_0), F_y(x_0,y_0)\rangle\cdot$
$\langle x-x_0, y-y_0\rangle$

Implicit Function

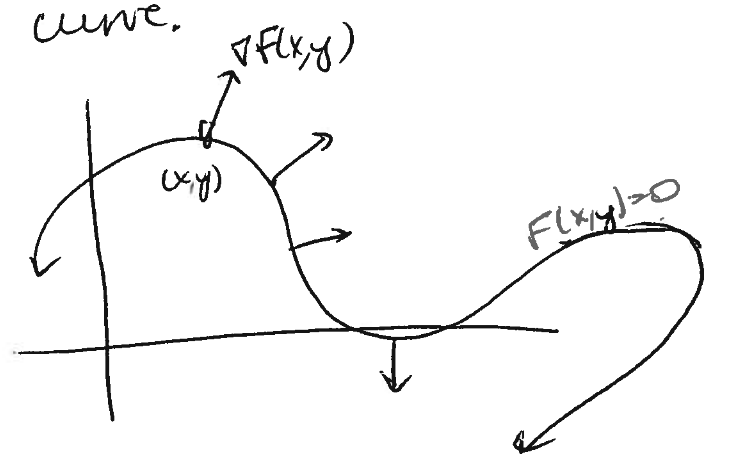
$F(x,y)=0$

full pt: plug in x_0, to solve for y_0
$F(x_0,y)=0$
(solve for y)

tangent line:

$$\boxed{\nabla F(x_0,y_0)\cdot\langle x-x_0, y-y_0\rangle=0}$$

Think about what the 2-d gradient is for a 2-d curve! It's a vector that's $\perp$ to the curve.



3-d

To find a tangent plane to a surface, at input pt (x_0, y_0).

Explicit function	Implicit Function
$z = f(x,y)$	$F(x,y,z) = 0$
full pt: $(x_0, y_0, f(x_0,y_0))$ i.e. $z_0 = f(x_0,y_0)$	full pt: plug in x_0 & y_0 to F + solve for z_0. $F(x_0, y_0, z) = 0$
tangent plane: $\boxed{z = z_0 + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)}$	tangent plane: $\boxed{\nabla F(x_0,y_0,z_0) \cdot \langle x-x_0, y-y_0, z-z_0\rangle = 0}$

$\Leftrightarrow$ $0 = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0) - (z-z_0)$

$\Leftrightarrow 0 = \langle f_x(x_0,y_0), f_y(x_0,y_0), -1\rangle \cdot \langle x-x_0, y-y_0, z-z_0\rangle$ (★)

If we let $F(x,y,z) = f(x,y) - z = 0$, then we get $F_x = f_x$, $F_y = f_y$, $F_z = -1$

$\Rightarrow$ (★) becomes

$0 = \langle F_x(x_0,y_0,z_0), F_y(x_0,y_0,z_0), F_z(x_0,y_0,z_0)\rangle \cdot \langle x-x_0, y-y_0, z-z_0\rangle$

$\Rightarrow 0 = \nabla F(x_0,y_0,z_0) \cdot \langle x-x_0, y-y_0, z-z_0\rangle$

In general, the only formulas you need:

(2-d) ① $\nabla F(x_0,y_0) \cdot \langle x-x_0, y-y_0\rangle = 0$

(3-d) ② $\nabla F(x_0,y_0,z_0) \cdot \langle x-x_0, y-y_0, z-z_0\rangle = 0$

What do you predict the formula will be for tangent hyperplane to a 5-d hypersurface?